

# TRUST AND SOCIAL COLLATERAL\*

DEAN KARLAN  
MARKUS MOBIUS  
TANYA ROSENBLAT  
ADAM SZEIDL

This paper builds a theory of trust based on informal contract enforcement in social networks. In our model, network connections between individuals can be used as social collateral to secure informal borrowing. We define network-based trust as the largest amount one agent can borrow from another agent and derive a reduced-form expression for this quantity, which we then use in three applications. (1) We predict that dense networks generate bonding social capital that allows transacting valuable assets, whereas loose networks create bridging social capital that improves access to cheap favors such as information. (2) For job recommendation networks, we show that strong ties between employers and trusted recommenders reduce asymmetric information about the quality of job candidates. (3) Using data from Peru, we show empirically that network-based trust predicts informal borrowing, and we structurally estimate and test our model.

## I. INTRODUCTION

A growing body of research demonstrates the importance of trust for economic outcomes.<sup>1</sup> Arrow (1974) calls trust “an important lubricant of a social system.” If trust is low, poverty can persist because individuals are unable to acquire capital, even if they have strong investment opportunities. If trust is high, informal transactions can be woven into daily life and help generate efficient allocations of resources. But what determines the level of trust between individuals?

In this paper we propose a model where the social network influences how much agents trust each other. Sociologists such as Granovetter (1985), Coleman (1988), and Putnam (2000) have long argued that social networks play an important role in

\*We thank Attila Ambrus, Susan Athey, Antoni Calvó-Armengol, Pablo Casas-Arce, Rachel Croson, Avinash Dixit, Drew Fudenberg, Andrea Galeotti, Ed Glaeser, Sanjeev Goyal, Daniel Hojman, Matthew Jackson, Rachel Kranton, Ariel Pakes, Andrea Prat, Michael Schwarz, Andrei Shleifer, Andy Skrzypacz, Fernando Vega-Redondo, and seminar participants for helpful comments. *dean.karlan@yale.edu*, *mobius@fas.harvard.edu*, *tanyar@iastate.edu*, *szeidl@econ.berkeley.edu*.

1. Trust has been linked with outcomes including economic growth (Knack and Keefer 1997), judicial efficiency and lack of corruption (La Porta et al. 1997), international trade and financial flows (Guiso, Sapienza, and Zingales 2009), and private investment (Bohnet, Herrman, and Zeckhauser 2008).

building trust.<sup>2</sup> In our model, networks create trust when agents use connections as social collateral to facilitate informal borrowing. The possibility of losing valuable friendships secures informal transactions in the same way that the possibility of losing physical collateral can secure formal lending.<sup>3</sup> Because both direct and indirect connections can serve as social collateral, the level of trust is determined by the structure of the entire network. Although we present our model in terms of trust in a borrowing transaction, it can also apply to other situations that involve moral hazard or asymmetric information, such as hiring workers through referrals.<sup>4</sup>

To understand the basic logic of our model, consider the examples in Figure I, where agent  $s$  would like to borrow an asset, such as a car, from agent  $t$ , in an economy with no formal contract enforcement. In Figure IA, the network consists only of  $s$  and  $t$ ; the value of their relationship, which represents either the social benefits of friendship or the present value of future transactions, is assumed to be 2. As in standard models of informal contracting,  $t$  will only lend the car if its value does not exceed the relationship value of 2. More interesting is Figure IB, where  $s$  and  $t$  have a common friend  $u$ , the value of the friendship between  $s$  and  $u$  is 3, and that between  $u$  and  $t$  is 4. Here, the common friend increases the borrowing limit by  $\min[3, 4] = 3$ , the weakest link on the path connecting borrower and lender through  $u$ , to a total of 5. The logic is that the intermediate agent  $u$  vouches for the borrower, acting as a guarantor of the loan transaction. If the borrower chooses not to return the car, he is breaking his promise of repayment to  $u$ , and therefore loses  $u$ 's friendship. Because the value of this friendship is 3, it can be used as collateral for a payment of up to 3. For the lender  $t$  to receive this amount,  $u$  must prefer transmitting the

2. Glaeser et al. (2000) show in experiments that social connections increase trust. Field evidence on the role of networks in trust-intensive exchange includes McMillan and Woodruff (1999) and Johnson, McMillan, and Woodruff (2002) for business transactions in Vietnam and transition countries; Townsend (1994) and Udry (1994) for insurance arrangements in India and Nigeria; and Macaulay (1963) and Uzzi (1999) for firms in the United States.

3. We abstract from morality, altruism, and other mechanisms that can generate trust even between strangers (e.g., Berg, Dickhaut, and McCabe [1995], Fukuyama [1995]); hence our definition of trust is like Hardin's (1992).

4. In related work, Kandori (1992), Greif (1993), and Ellison (1994) develop models of community enforcement where deviators are punished by all members of society. More recently, Dixit (2003), Lippert and Spagnolo (2006), Ali and Miller (2008), and Bloch, Genicot, and Ray (2008) have explored models of informal contracting where networks are used to transmit information. In contrast, in our work the network serves as social collateral.

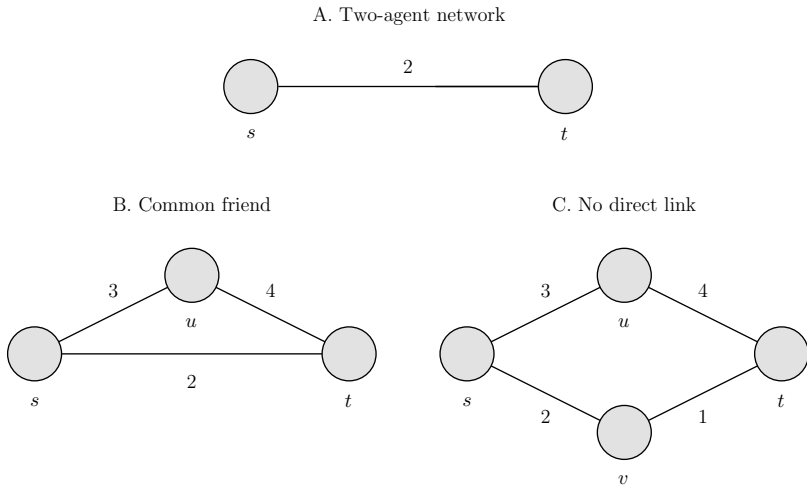


FIGURE I  
Social Collateral in Simple Networks

This figure illustrates the calculation of trust in simple networks. In all three panels, agent  $s$  wishes to borrow an asset from agent  $t$ . In Panel A, both agents are direct friends and the borrowing limit is equal to 2, the strength of their relationship. In Panel B, their relationship is strengthened by a common friend  $u$  and the borrowing limit increases by  $\min[3, 4] = 3$ , which is the value of the weakest link on the path connecting  $s$  and  $t$  through  $u$ . In Panel C, borrower and lender are not direct friends and the borrowing limit is the sum of the weakest links on the two paths between  $s$  and  $t$ . See the text for details.

payment to losing the friendship with him, explaining the role of the weakest link.

Our main theoretical result is that in general networks, the level of trust equals the sum of the weakest link values over all disjoint paths connecting borrower and lender. This quantity is called the *maximum network flow*, a well-studied concept in graph theory.<sup>5</sup> Intuitively, the maximum flow is the largest amount that can flow from borrower to lender along the edges of the network, respecting the capacity constraints given by link values. This concept does not require the borrower and the lender to be directly linked; for example, in Figure IC, where  $s$  and  $t$  are not connected but share two common friends, the borrowing limit is the sum of the weakest links on the two paths connecting  $s$  and  $t$ ,  $\min[3, 4] + \min[2, 1] = 4$ , because each intermediate agent can vouch for part of the value of the car. The key idea in the proof of our main result is to characterize coalition-proof informal

5. See Cormen et al. (2001) for a textbook treatment.

contracts using the maximum flow–minimum cut theorem (Ford and Fulkerson 1956), a famous result in computer science.

The paper also develops three applications of this social collateral model. The first application, which explores the effect of network structure on welfare, helps reconcile two seemingly competing views of sociologists. Coleman (1988) emphasizes the benefits of networks with high closure, where connected agents share many common friends, which facilitate the enforcement of cooperation. In contrast, Burt (1995) argues that loose networks, that is, low closure, are better, because they provide greater access to information and other resources. The social collateral model can reconcile these views by identifying a trade-off between trust and access, which implies that the relative benefit of high or low closure depends on the value of the assets being transacted. Closure is more attractive when agents tend to exchange valuable assets, because it maximizes trust among a small number of individuals. This is in line both with Coleman's general argument and with his example of diamond dealers in New York, who exchange valuable stones in a tight network of family and religious ties. In contrast, when the network is mainly used to exchange small favors such as giving information or advice, large and loose neighborhoods are better because they maximize access to these resources. These results also provide foundations and network-based measures for Putnam's (2000) concepts of bonding versus bridging social capital and have implications for the design of organizations.

In a second application, we study the implications of network-based trust for job recommendations. It is well known that many jobs are found through social networks (Ioannides and Loury 2004). A common explanation is that information about job openings spreads through friends and acquaintances. This "strength of weak ties" argument, made by Granovetter (1973), predicts that weak links to agents with whom one has few common friends are most useful for job search, because they provide access to otherwise unobtainable information. However, the evidence is mixed: many studies find that strong ties in dense networks are more important.

Our model suggests a reason for the strength of strong ties in job search: trusted recommenders can reduce asymmetric information about job candidates. In the social collateral model, networks do help identify high-type workers, but only if the trust flow between the recommenders and the employer exceeds the sensitivity of profits to worker type. Recommendations from less

trusted individuals are not credible, because a low-type candidate can bribe the recommender to put in a good word for him. This result implies that the relative importance of weak versus strong connections should vary as a function of the skill sensitivity of the job, which can help explain the mixed evidence about weak ties. We also obtain new predictions: agents hired through the network should earn higher wages; this wage gap should be increasing in the skill intensity of the job; and employers should rely more on social networks to fill skill-intensive vacancies. Although these predictions do not emerge in a model of information transmission about vacancies, they are consistent with existing evidence, suggesting that trusted referrals can be important for understanding job search.

In the third, empirical application, we estimate and test our model using a unique data set on social networks and informal lending in two low-income shantytowns in Peru. In these communities, informal borrowing is very common, making the data an ideal fit for our theory. For example, 46% of households have recently borrowed money from others in their immediate social networks. We estimate the social collateral model in this data using a discrete choice framework, which allows us to back out the relative strength of network links as a function of time spent together, and establish three results. (1) Confirming our main prediction, we document a strong positive correlation between social collateral and borrowing, which is primarily driven by strong ties. For example, increasing trust flow by a link in the top one-third of the distribution of time spent together increases the probability of borrowing by a factor of 2.7. (2) We show that direct and indirect paths have similar effects on borrowing, demonstrating the importance of network closure for building trust. (3) We verify the key structural implication that borrowing should be determined by the weakest link on a path. Our results are inconsistent with alternative explanations such as altruism or information transmission, which do not predict that indirect paths should matter through the value of the weakest link. Taken together, we find strong support for the social collateral model; our results also suggest that strong ties and high closure, i.e., bonding social capital, are particularly important for borrowing.

The rest of the paper is organized as follows. Section II collects motivating evidence about the social collateral mechanism. Section III develops the model and derives the reduced form expression for trust. Section IV presents our theoretical applications

and Section V our empirical application. Section VI concludes by sketching some other applications. All proofs are in the Appendix.

## II. SOCIAL COLLATERAL: SUGGESTIVE EVIDENCE

This section presents evidence about social networks and informal contract enforcement. It is a well-documented fact that social networks are often used for trust-intensive exchanges.<sup>6</sup> In this section, we focus on documenting anecdotal evidence about the *mechanism* through which networks create the trust necessary for these transactions.

We begin with an example originally attributed to Wechsberg (1966), which we take from Coleman (1990). This example is of a prominent Norwegian shipowner who was in need of a ship that had undergone repairs in an Amsterdam shipyard. However, “the yard would not release the ship unless a cash payment was made of 200,000 pounds. Otherwise the ship would be tied up for the weekend, and the owner would lose at least twenty thousand pounds.” The shipowner was in trouble, because he could not have 200,000 pounds delivered immediately to Amsterdam. To solve this problem, he called a London banker at Hambros, who presumably had contacts in Amsterdam. After hearing the situation, “the Hambros man looked at the clock and said, ‘It’s getting late but I’ll see whether I can catch anyone at the bank in Amsterdam . . . stay at the phone.’ Over a second phone he dictated to a secretary in the bank a telex message to the Amsterdam bank: ‘Please pay 200,000 pounds telephonically to (name) shipyard on understanding that (name of ship) will be released at once.’”

In this example, the shipowner borrowed 200,000 pounds on immediate notice from an Amsterdam bank with which he had no direct connection. He accomplished this by collateralizing two business relations: his connection with the London banker, and the connection between the London and Amsterdam banks. In Coleman’s (1990) terminology, the London banker acted as a “trust intermediary”: by vouching for the shipowner, he provided access and created the necessary trust for the transaction. If the shipowner were to default, the Amsterdam bank could ask the London banker to pay compensation or risk jeopardizing their relationship; and similarly, the London banker could presumably

6. For references, see the citations in footnote 2, as well as Table 1 in our working paper, Mobius and Szeidl (2007).

extract money from the shipowner if needed. This is how the two business relations were used as collateral to secure borrowing.

A second example of the mechanism through which networks generate trust is the *guanxi* system in China. *Guanxi* refers to a trusted relationship that can be used to obtain services either directly or indirectly from a person's social network.<sup>7</sup> *Guanxi* often serves as a substitute for legal contracts, and helps overcome institutional weaknesses of the Chinese legal system (Fock and Woo 1998). To understand the mechanism of *guanxi*, consider Standifird and Marshall's (2000) example of a buyer and a seller who share *guanxi* with a common acquaintance. This third person can act as *zhongjian ren*, essentially an intermediary, by introducing the buyer to the supplier. In this transaction, the *zhongjian ren* vouches for the buyer by assuring the supplier that he will be compensated for any sunk investments required for the relationship (e.g., preparing blueprints or samples). If the buyer exploits the supplier, the intermediary will be held responsible; and unless reparations are made, this can damage the relationship between the intermediary and either business partner. This example illustrates the collateral role of *guanxi*: parties refrain from cheating because it would limit their future exchanges with the intermediary whose *guanxi* they borrowed.

Both of these examples highlight the role of vouching intermediaries and the collateral function of connections in securing transactions. We now develop a model that formalizes these ideas.

### III. THEORY

This section presents a game-theoretic model of informal borrowing in social networks, and shows that the highest loan amount is limited by the *maximum network flow* (or *trust flow*) between borrower and lender. In Sections IV and V, where we consider applications, we make use of this reduced form characterization of trust.

#### III.A. Model Setup

In our model, a borrower needs an asset of a lender to produce a social surplus. This asset might represent a factor of production,

7. The original meaning of the phrase "guan-xi" is using relationships to gain indirect access to a wider network. "Guan" means gate or hurdle and "xi" refers to a relationship; *guanxi* is thus a gateway to other relationships.

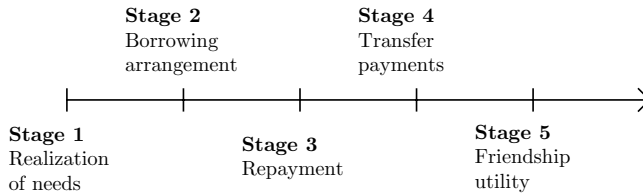


FIGURE II  
Model Timeline

such as a farming tool, a vehicle, or an animal; it could also be an apartment, a household durable good, or simply a cash payment. In the absence of legal contract enforcement, borrowing must be secured by an informal arrangement. In our model, the social network is used for this purpose: connections in the network have associated consumption value, which serve as “social collateral” to enable borrowing.

Formally, a social network  $G = (W, E)$  consists of a set  $W$  of agents (vertices or nodes) and a set  $E$  of edges (links), where an edge is an unordered pair of distinct vertices. Each link in the network represents a friendship or business relationship between the two parties involved. We formalize the strength of relationships using an exogenously given capacity  $c(u, v)$ .

**DEFINITION 1.** A capacity is a function  $c : W \times W \rightarrow \mathbb{R}$  such that  $c(u, v) > 0$  if  $(u, v) \in E$  and  $c(u, v) = 0$  otherwise.

The capacity measures the utility benefits that agents derive from their relationships. For ease of presentation, we assume that the strength of relationships is symmetric, so that  $c(u, v) = c(v, u)$  for all  $u$  and  $v$ .<sup>8</sup>

Our model consists of five stages, which are depicted in Figure II. We begin by describing the model and then discuss the economic content of our modeling assumptions.

*Stage 1: Realization of Needs.* Two agents  $s$  and  $t$  are randomly selected from the social network. Agent  $t$ , the lender, has an asset that agent  $s$ , the borrower, desires. The lender values the asset at  $V$ , and it is assumed that  $V$  is drawn from some prior distribution  $F$  over  $[0, \infty)$ . The identity of the borrower and the lender and the value of  $V$  are publicly observed by all players.

8. Our results extend to the case where capacities are asymmetric. In that environment, the social network can be represented as a directed graph and the directed network flow determines borrowing.



*Stage 2: Borrowing Arrangement.* At this stage, the borrower publicly proposes a *transfer arrangement* to all agents in the social network. The role of this arrangement is to punish the borrower and compensate the lender in the event of default. A transfer arrangement consists of a set of transfer payments  $h(u, v)$  for all  $u$  and  $v$  agents involved in the arrangement. Here  $h(u, v)$  is the amount  $u$  promises to pay  $v$  if the borrower fails to return the asset to the lender. Once the borrower has announced the arrangement, all agents involved have the opportunity to accept or decline. If all involved agents accept, then the asset is borrowed and the borrower earns an income  $\omega(V)$ , where  $\omega$  is a nondecreasing function with  $\omega(0) = 0$ . If some agents decline, then the asset is not lent, and the game moves on directly to stage 5.

*Stage 3: Repayment.* Once the borrower has made use of the asset, he can either return it to the lender or steal it and sell it for a price of  $V$ .<sup>9</sup> If the borrower returns the asset, then the game moves to the final stage 5.

*Stage 4: Transfer Payments.* All agents observe whether the asset was returned in the previous stage. If the borrower did not return the asset, then the transfer arrangement is activated. Each agent has a binary choice: either he makes the promised payment  $h(u, v)$  in full or he pays nothing. If some agent  $u$  fails to make a prescribed transfer  $h(u, v)$  to  $v$ , then he loses his friendship with agent  $v$  (i.e., the  $(u, v)$  link “goes bad”). If  $(u, v)$  link is lost, then the associated capacity is set to zero for the remainder of the game. We let  $\tilde{c}(u, v)$  denote the new link capacities after these changes.

*Stage 5: Friendship Utility.* At this stage, agents derive utility from their remaining friends. The total utility enjoyed by an agent  $u$  from his remaining friends is simply the sum of the values of all remaining relationships, that is,  $\sum_v \tilde{c}(u, v)$ .

Our model is a multistage game with observed actions. Let  $\Sigma_u$  denote the set of agent  $u$ 's pure strategies and let  $\Sigma = \times_u \Sigma_u$ . We focus on the set of pure strategy subgame perfect equilibria below.

### III.B. Discussion of Modeling Assumptions

We now discuss some of the assumptions underlying our model.

9. As we show in Appendix II, the model can be extended to the case where the liquidation value of the asset is  $\phi \cdot V$  with  $\phi \leq 1$ .

*Social Sanctions.* When an agent fails to make a promised transfer, we assume that the associated friendship link automatically goes bad, capturing the idea that friendly feelings often cease to exist if a promise is broken. Appendix II develops explicit micro foundations for this assumption. In these micro foundations, which build on Dixit (2003), failure to make a transfer is a signal that the agent no longer values his friend, in which case these former friends find it optimal not to interact with each other in the future. An alternative justification is that people break a link for emotional or instinctive reasons when a promise is not kept; Fehr and Gächter (2000) provide evidence for such behavior.

*Circle of Trust.* For large social networks it can be unrealistic for the borrower to include socially distant agents in the arrangement. All our results hold if we restrict the set of links over which transfer payments can be proposed to some subgraph of the original network, the “circle of trust,” which may depend on the identity of the borrower and the lender. The only difference in our results is that the network flow measure of the borrowing limit will have to be computed in the subgraph of permissible links.

*Transfer Arrangement as Social Norms.* The transfer arrangement in our model can be interpreted either as an explicit agreement between all parties or as a representation of accepted norms of behavior. In the second interpretation, agents simply share an understanding about what they are expected to do in the event of default.

*Cash Bonds and Borrowing Constraints.* One way to solve the moral hazard problem is to have the borrower post a cash bond to the lender, which is returned only if the borrower does not default on the asset loan. We abstract away from bonds and prepayments by assuming that the borrower is initially cash-constrained. However, we do allow the borrower and other agents to make payments in later stages of the game. This can be justified if agents work or make investments in the initial stage, generating income in later stages; or if transfers are in-kind, for example, helping out with the harvest, where posting a bond may be inefficient or infeasible.

### *III.C. Equilibrium Analysis*

For what values of  $V$  can borrowing be implemented in a subgame perfect equilibrium? We begin answering this question

by studying equilibria where all promises are kept, that is, where every transfer  $h(u, v)$  is expected to be paid if the borrower fails to return the asset. We later show that focusing on these equilibria is without loss of generality. In any equilibrium where promises are kept, transfers have to satisfy the capacity constraint

$$(1) \quad h(u, v) \leq c(u, v).$$

This is simply an incentive compatibility constraint. If the borrower fails to return the asset, agent  $u$  has to decide whether to make his promised transfer payment  $h(u, v)$  to  $v$ . The cost of making the payment is  $h(u, v)$ ; the cost of not making the payment is  $c(u, v)$ , because it results in losing the friendship with  $v$ . In any equilibrium where promises are kept,  $u$  must prefer the friendship over the monetary value of the transfer, leading to (1).

*Two-Agent Network.* To build intuition, we begin the equilibrium analysis with the case where the social network consists only of the borrower  $s$  and the lender  $t$ . Let  $\sigma$  be a pure strategy subgame perfect equilibrium that implements borrowing where promises are kept. In any such equilibrium,  $V \leq h(s, t)$ . To see why, assume that the borrower  $s$  defaults on the equilibrium path. Then the lender receives the transfer payment  $h(s, t)$  instead of the asset; but he must break even to lend, which yields  $V \leq h(s, t)$ . On the other hand, if the borrower returns the asset on the equilibrium path, then he must weakly prefer not to default, which again requires  $V \leq h(s, t)$ . Combining this inequality with the capacity constraint (1) then yields

$$(2) \quad V \leq c(s, t),$$

showing that borrowing is limited by the maximum flow in this simple network. It is also easy to see that when (2) is satisfied, there exists an equilibrium that implements borrowing: just set  $h(s, t) = V$ .<sup>10</sup> Intuitively, the collateral value of friendship can be used to elicit payment and thus solve the agency problem.

*Four-Agent Network.* To gain intuition about borrowing in more general networks, we next consider the network depicted

10. In this equilibrium, all surplus accumulates to the borrower because of our assumption that he proposes the transfer arrangement. In a setup where bargaining power is more evenly distributed, we expect that the surplus would be shared by the agents involved in the transfer arrangements, in a manner similar to Goyal and Vega-Redondo (2007).

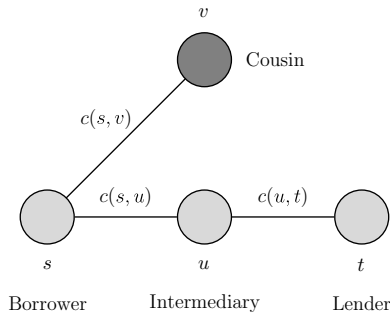


FIGURE III  
Borrowing in a Four-Agent Network

This figure illustrates borrowing in networks with intermediaries. The arrangement favored in our paper involves transfers flowing from  $s$  through  $u$  to  $t$  in the event of default. In this arrangement the weakest link,  $\min[c(s, u), c(u, t)]$ , determines the borrowing limit. An alternative arrangement, where cousin  $v$  promises to punish the borrower  $s$  in case of default, sometimes enforces better outcomes. However, this arrangement is not robust to side deals by groups of agents: the borrower and his cousin can jointly deviate, steal the asset, and short-change the lender. As we show in the text, all side deal-proof arrangements satisfy the weakest link requirement.

in Figure III, which consists of four players: the borrower  $s$ , the lender  $t$ , an intermediate agent  $u$  connecting  $s$  and  $t$ , and an agent  $v$  who is connected only to the borrower  $s$ . We will refer to  $v$  as the “cousin” of  $s$ . A natural transfer arrangement that implements borrowing in this network is one where agent  $u$  acts as an intermediary who elicits and transits payments from  $s$  to  $t$  in the case of no compliance, and gets zero net profits. To formalize this arrangement, simply set  $h(s, u) = h(u, t) = V$ . For this arrangement to be incentive compatible, the capacity constraint (1) must be satisfied for both links involved:  $V \leq c(s, u)$  must hold so that  $s$  delivers the transfer to  $u$ , and  $V \leq c(u, t)$  is needed to ensure that  $u$  passes on the transfer to  $t$ . Combining these yields the “weakest link” inequality

$$(3) \quad V \leq \min [c(s, u), c(u, t)],$$

which implies that the maximum flow determines the borrowing limit in this transfer arrangement.

However, networks with more than two agents generally admit other subgame perfect equilibria that can implement borrowing even if (3) fails. We argue that these equilibria are implausible, because they fail a natural coalition-proofness requirement. To illustrate, assume that the borrower  $s$  has a strong link to his cousin

$v$ , with a capacity value of  $c(s, v) = V + 1$ . The borrower might then propose an informal arrangement in which he promises to pay his cousin a transfer of  $h(s, v) = V + 1$  in case he fails to return the asset. This arrangement provides the right incentives to the borrower, and is a subgame perfect equilibrium even if (3) fails. To understand its logic, note that in this arrangement, the borrower essentially makes the following proposal to the lender: “Lend me your asset; if I don’t return it to you, my cousin will be angry with me.” As this interpretation makes it clear, this borrowing arrangement may not be robust to joint deviations where both the borrower and his cousin depart from equilibrium. More concretely, the borrower could circumvent the arrangement by entering a *side deal* with his cousin, in which he steals the asset and shares the proceeds with the cousin (who in equilibrium would otherwise receive nothing). Due to the possibility of such side deals, we do not find this equilibrium plausible.

A similar potential equilibrium is one where the intermediate agent  $u$  provides incentives to the borrower but promises a zero transfer to the lender. In this case, the lender effectively “outsources” monitoring to the intermediary, trusting that the borrower will always return the asset rather than pay a high transfer to  $u$ . This arrangement is again open to side deals: here  $s$  and  $u$  can choose to steal the asset jointly and split the proceeds, leaving the lender with nothing. As in the equilibrium with the cousin, the possibility of a side deal arises because nobody “monitors the monitor”: the lender is not fully in control of incentives. When enforcement is outsourced to either the cousin or the intermediary, these agents can team up with the borrower and steal the asset.

These examples suggest that when the borrower and other agents can agree to side deals, it may not be in the interest of the lender to provide the asset. This motivates our focus on subgame perfect equilibria that are immune to such side deals.

### III.D. Side Deal–Proof Equilibrium

Consider the subgame starting in stage 2, after the identities of the borrower and the lender and the value of the asset are realized, and for any pure strategy  $\sigma \in \Sigma$ , let  $U_u(\sigma)$  denote the total utility of agent  $u$  in this subgame. We formalize the idea of a side deal as an alternative transfer arrangement  $\tilde{h}(u, v)$  that  $s$  proposes to a subset of agents  $S \subset W$  after the original arrangement is accepted. If this side deal is accepted, agents in  $S$  are expected to make transfer payments according to  $\tilde{h}$ , whereas agents outside

$S$  continue to make payments described by  $h$ . For the side deal to be credible to all participating agents, it must be accompanied by a proposed path of play that these agents find optimal to follow. This motivates the following definition.

**DEFINITION 2.** A side deal with respect to a pure strategy profile  $\sigma$  is a set of agents  $S$ , a transfer arrangement  $\tilde{h}(u, v)$  for all  $u, v \in S$ , and a set of continuation strategies  $\{\tilde{\sigma}_u \mid u \in S\}$  proposed by  $s$  to agents in  $S$  at the end of stage 2, such that

- (i)  $U_u(\tilde{\sigma}_u, \tilde{\sigma}_{S \setminus u}, \sigma_{-S}) \geq U_u(\sigma'_u, \tilde{\sigma}_{S \setminus u}, \sigma_{-S})$  for all  $\sigma'_u$  and all  $u \in S$ ,
- (ii)  $U_u(\tilde{\sigma}_S, \sigma_{-S}) \geq U_u(\sigma_S, \sigma_{-S})$  for all  $u \in S$ , and
- (iii)  $U_s(\tilde{\sigma}_S, \sigma_{-S}) > U_s(\sigma_S, \sigma_{-S})$ .

Condition (i) says that all agents  $u$  involved in the side deal are best-responding on the new path of play, that is, that the proposed path of play is an equilibrium for all agents in  $S$  conditional on others playing their original strategies  $\sigma_{-S}$ . Condition (ii) says that if any agent  $u \in S$  refuses to participate in the side deal, then play reverts to the original path of play given by  $\sigma$ . Finally, (iii) ensures that the borrower  $s$  strictly benefits from the side deal.

**DEFINITION 3.** A pure strategy profile  $\sigma$  is a side deal–proof equilibrium if it is a subgame perfect equilibrium that admits no side deals.

It is easy to see that this condition rules out the equilibria violating the weakest link inequality (3) in Figure III. We now turn to extend this result to general networks.<sup>11</sup>

### III.E. Main Theorem

We begin by formally defining the concept of network flows intuitively discussed above.

**DEFINITION 4.** An  $s \rightarrow t$  flow with respect to capacity  $c$  is a function  $f : G \times G \rightarrow \mathbb{R}$  that satisfies the following:

- (i) Skew symmetry:  $f(u, v) = -f(v, u)$ .
- (ii) Capacity constraints:  $f(u, v) \leq c(u, v)$ .
- (iii) Flow conservation:  $\sum_w f(u, w) = 0$  unless  $u = s$  or  $u = t$ .

11. Our definition of side deal–proof equilibrium does not require side deals to be robust to further side deals. However, as the proof in Appendix I makes clear, imposing this requirement would not change any of our results: when there is a deviating side deal, there is also one that is robust to further coalitional deviations, namely the side deal implemented with a network flow.

The value of a flow is the amount that leaves the borrower  $s$ , given by  $|f| = \sum_w f(s, w)$ . Let  $T^{st}(c)$  denote the maximum value among all  $s \rightarrow t$  flows.

**THEOREM 1.** There exists a side deal–proof equilibrium that implements borrowing between  $s$  and  $t$  if and only if the asset value  $V$  satisfies

$$(4) \quad V \leq T^{st}(c).$$

This result states that the endogenous borrowing limit equals the value of the maximum flow between borrower  $s$  and lender  $t$ . We interpret the borrowing limit  $T^{st}(c)$  as a measure of network-based trust: if  $s$  can borrow more from  $t$ , it must be that  $t$  has higher trust in  $s$ .

The logic of the proof of Theorem 1 is as follows. When  $V$  satisfies inequality (4), a side deal–proof equilibrium is easy to construct: by assumption, there exists an  $s \rightarrow t$  flow with value  $V$ , and this flow can be used as a transfer arrangement. Flow conservation implies that all intermediate agents break even, confining their role to simply extracting and transmitting the payment  $V$  from  $s$  to  $t$  in case  $s$  fails to return the asset. Thus the lender is in full control of incentives; because of this, the equilibrium is easily seen to be side deal–proof.

To show that no side deal–proof equilibrium can implement a higher level of borrowing, we build on the maximum flow–minimum cut theorem (Ford and Fulkerson 1956), which states that the maximum network flow between  $s$  and  $t$  equals the value of the minimum cut. A cut is a disjoint partition of the nodes into two sets  $G = S \cup T$  such that  $s \in S$  and  $t \in T$ , and the value of the cut is defined as the sum of  $c(u, v)$  for all links such that  $u \in S$  and  $v \in T$ .

For any borrowing arrangement violating (4), we can construct a side deal in the following way. Fix a minimum cut  $(S, T)$ ; the maximum flow–minimum cut theorem implies that the total capacity of all links between  $S$  and  $T$  is less than  $V$ . But then agents in  $S$  have a profitable side deal: by defecting as a group, they lose less than  $V$  in foregone friendships, but gain  $V$  from selling the asset. For a concrete example, consider the network with the cousin in Figure III and suppose that  $c(s, u) < c(u, t)$ . The minimum cut between  $s$  and  $t$  has value  $c(s, u)$ , and the corresponding partition is simply  $S = (s, v)$  and  $T = (u, t)$ . In any equilibrium where  $V > c(s, u)$ , that is, where (4) is violated, agents in

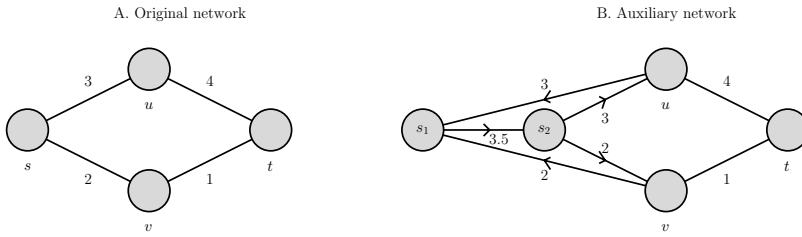


FIGURE IV  
Maximum Network Flow with Transfer Constraints

This figure illustrates network flow with transfer constraints. Agent  $s$  could normally borrow an asset up to value 4 from agent  $t$ . However, he faces a binding transfer constraint of 3.5. We can calculate network flow in the constrained graph by drawing an auxiliary network where we split  $s$  into two agents  $s_1$  and  $s_2$ . All incoming links of agent  $s$  are connected to  $s_1$  and all outgoing links emanate from agent  $s_2$ . A directed link from  $s_1$  to  $s_2$  has capacity equal to the transfer constraint. The network flow from  $s_1$  to agent  $t$  equals the maximum network flow with transfer constraint.

$S$  have a side deal: the borrower  $s$  and his cousin  $v$  can team up to steal the asset, because their total repayment is limited by the value of the cut  $c(s, u)$ .

### III.F. Extensions: Transfer Constraints and Endogenous Circle of Trust

*Transfer Constraints.* In environments with credit constraints, agents might have limits on the total amount they can borrow or transfer. For example, in Figure IVA, the intermediaries  $u$  and  $v$  might worry that if the borrower  $s$  carried too large a debt burden, he would be unable to pay. We show that the concept of network flows can be used to characterize borrowing in this environment as well. To introduce borrowing and transfer constraints in a simple way, suppose that each agent  $u$  can make a total payment of at most  $k_u$  to others in the network, where the transfer constraints  $k_u$  are exogenous. Here  $k_u$  can represent either cash or time constraints.<sup>12</sup>

How much borrowing can be implemented in this environment? We show that the answer is given by the maximum flow in a modification of the social network, where each agent  $u$  is replaced by two identical agents connected by a link with capacity

12. For an intermediate agent (but not for the borrower), incoming transfers may help relax cash constraints. For these intermediate agents,  $k_u$  represents constraints that remain after incoming payments; for example, these could be time constraints if the transfers were in-kind services, such as helping out, which cannot be easily passed on.



$k_u$ . To formally construct this auxiliary (directed) network  $G'$ , replace each node  $u$  in  $G$  with a pair of nodes,  $u_1$  and  $u_2$ , and replace each  $(u, v)$  link with two new directed links, a  $u_2 \rightarrow v_1$  link and a  $v_2 \rightarrow u_1$  link, each with capacity equal to  $c(u, v)$ . Finally, for each agent  $u$ , create a new  $u_1 \rightarrow u_2$  link with capacity equal to the transfer constraint  $c(u_1, u_2) = k_u$ . That is, we duplicate all agents  $u$ , point all incoming links of  $u$  to  $u_1$ , have all outgoing links of  $u$  originate in  $u_2$ , and let the capacity of the  $u_1 \rightarrow u_2$  link be  $k_u$ .

For example, consider the network in Figure IVA, where agent  $s$  faces a binding transfer constraint of 3.5. The corresponding auxiliary network is drawn in Figure IVB and we can deduce that the constrained network flow equals 3.5, the flow from agent  $s_1$  to agent  $t$  in the auxiliary graph.

In Appendix I we show that in any side deal–proof equilibrium where promises are kept, the borrowing limit in the presence of transfer constraints equals the value of the maximum  $s_1 \rightarrow t_1$  flow in  $G'$ . To understand the intuition, consider a maximal flow. As in the basic model, the amounts assigned to links between agents by this flow can be interpreted as the transfer payments in a candidate transfer arrangement. It remains to verify that, in this arrangement, no agent  $u$  exceeds his total transfer constraint  $k_u$ . But this follows by construction of  $G'$ . The total transfers promised by  $u$  must be equal to the flow leaving  $u_2$  in  $G'$ ; but by flow conservation, this must be equal to the value carried over the  $u_1 \rightarrow u_2$  link, which is bounded by the link capacity of  $k_u$  in  $G'$ .

*Circle of Trust.* We can endogenize the “circle of trust,” that is, the set of permissible links over which transfer arrangements can be proposed, by assuming that there is a fixed cost associated with proposing various transfer arrangements. For each subgraph  $G_0 \subseteq G$ , let  $\kappa(G_0) \geq 0$  denote the cost of a transfer arrangement that includes all links in  $G_0$ .<sup>13</sup> Assume that  $\kappa$  is monotone in the sense that if  $G_0 \subseteq G'_0$  then  $\kappa(G_0) \leq \kappa(G'_0)$ . The function  $\kappa$  can be interpreted as a characteristic of the community’s social norm; for example, in a kin-based society, we expect  $\kappa$  to be zero or small for most family and relative links.

Agent  $s$ , who wishes to borrow  $V$  from  $t$ , must now solve the cost-minimization problem  $\min\{\kappa(G_0) \mid G_0 \subseteq G \text{ such that } T_{G_0}^{st} \geq V\}$ , where  $T_{G_0}^{st}$  is the trust flow between  $s$  and  $t$  in  $G_0$ . The solution

13. For two networks  $G = (W, E)$  and  $G' = (W', E')$ , we say that  $G' \subseteq G$  if  $W' \subseteq W$  and  $E' \subseteq E$ .

$G_0^*$ , if it exists, is the minimum cost subgraph where borrowing  $V$  can still be supported. Agent  $s$  then chooses to borrow if and only if his profit from the loan exceeds the cost, that is,  $\omega(V) \geq \kappa(G_0^*)$ . Besides its added flexibility, this framework also yields two new implications. (1) The set of people involved in an arrangement is endogenously determined: the greater the profits  $\omega(V)$ , the more the borrower is willing to extend his circle of trust.<sup>14</sup> (2) With positive  $\kappa$ , agents only borrow when profits are high enough; assets that generate low returns are never secured through social collateral.

#### IV. APPLICATIONS

##### IV.A. Network Structure and Welfare

We now explore how the network structure affects the payoffs from borrowing in the social collateral model. Because the network is completely summarized by the vector of capacities  $c$ , the borrowing limit  $T^{st}(c)$  can be viewed as a “trust map” that determines, as a function of the network structure  $c$ , how much trust is created between  $s$  and  $t$ . To see how trust determines payoffs, let  $\Pi^{st}(c)$  denote the expected payoff of  $s$  from borrowing, conditional on the lender being agent  $t$ ; then

$$(5) \quad \Pi^{st}(c) = \Pi(T^{st}(c)), \text{ where } \Pi(z) = \int_0^z \omega(v) dF(v),$$

because the payoff is just the expectation of  $\omega(V)$  over all values of  $V$  that do not exceed the borrowing limit  $T^{st}(c)$ . Changes in the network affect the payoffs  $\Pi$  through changes in the trust flow  $T^{st}(c)$ . Our goal in this section is to characterize these welfare effects.<sup>15</sup>

*Monotonicity.* We first explore the effect of increasing connectivity by adding new links or strengthening existing links. We say that the network associated with capacity  $c_1$  is *more strongly connected* than that associated with  $c_2$  if no link has lower capacity under  $c_1$  than under  $c_2$ ; that is,  $c_1(u, v) \geq c_2(u, v)$  for all  $u, v \in W$ . We then have the following monotonicity result.

14. Formally, an increase in  $\omega(V)$  holding fixed  $V$  can change the sign of  $\omega(V) - \kappa(G_0^*)$  from negative to positive and induce borrowing.

15. Besides the profit from borrowing  $\Pi^{st}(c)$ , the borrower  $s$  also derives utility from his friends. In the subsequent analysis we focus on the payoff from borrowing.

PROPOSITION 1. If the social network with capacity  $c_1$  is more strongly connected than the network with capacity  $c_2$ , then for any borrower  $s$  and lender  $t$ , both trust and payoffs are higher:  $T^{st}(c_1) \geq T^{st}(c_2)$  and  $\Pi^{st}(c_1) \geq \Pi^{st}(c_2)$ .

Networks with more and stronger links generate more trust and higher payoffs due to the increased supply of social collateral. A large body of work in sociology relies on the result formalized here: Putnam's (1995), for example, argues that "networks of civic engagement (...) encourage the emergence of social trust." The fact that this monotonicity emerges naturally in the social collateral model makes it a useful candidate for exploring other questions related to network-based trust.

*Closure and Structural Holes.* We now turn to study how the deeper structure of the network affects payoffs, focusing on changes in network closure, a concept often discussed in the sociology literature. Networks have high closure if the neighborhoods of connected agents have a large overlap. To illustrate, consider the two network neighborhoods of agent  $s$  in Figure V, which is a small variation of Figure 1 in Coleman (1988). The neighborhood of  $s$  in Figure VB has higher closure, because the friends of  $s$  are directly connected. This idea of closure can also be formulated using network paths: a neighborhood has high closure if it connects  $s$  to few others through many paths (as in Figure VB), whereas it has low closure if it connects  $s$  to many others through fewer paths each (Figure VA).

The sociological literature has two views about the benefits of closure. One view, dating back to Coleman (1988), argues that high closure is good because it facilitates sanctions, making it easier for individuals to trust each other. In his discussion of the wholesale diamond market in New York City, Coleman explains that "If any member of this community defected through substituting other stones or stealing stones in his temporary possession, he would lose family, religious and community ties." Similarly, in the context of Figure V, Coleman argues that in the high closure network of Figure VB, agents  $t_1$  and  $t_2$  can "combine to provide a collective sanction, or either can reward the other for sanctioning."

In contrast, Granovetter (1973) and Burt (1995) argue that loose networks with low closure lead to higher performance, because they allow agents to reach many others through the network. Burt also emphasizes the role of structural holes, that is, people who bridge otherwise disconnected networks: for example,

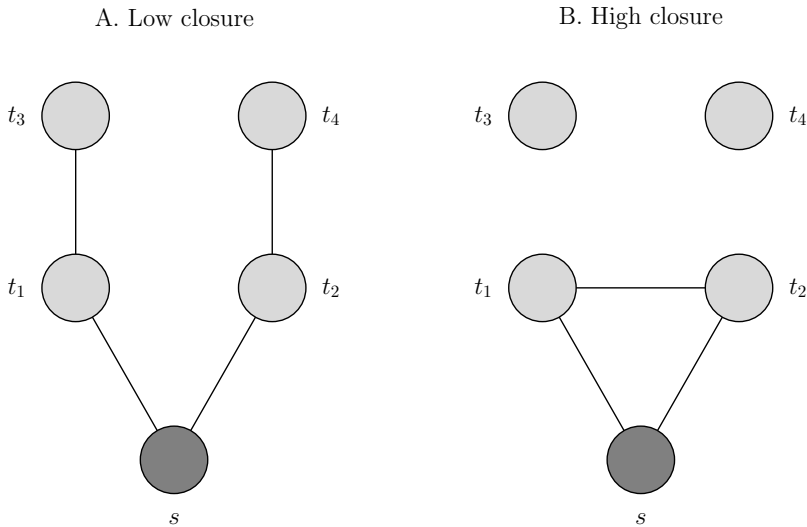


FIGURE V

## Network Neighborhoods with Increasing Network Closure

This figure shows network neighborhoods with increasing network closure. The two neighborhoods shown are a small variation on Figure 1 in Coleman (1988). With unit link capacities, agent  $s$  is connected through four paths to the rest of the network in both neighborhoods. In a low-value exchange environment, the neighborhood in Panel A is more attractive because it provides access to more people. In a high-value-exchange environment, the neighborhood in Panel B is more attractive, because closure allows borrowing high-valued assets from  $t_1$  and  $t_2$ .

$s$  is a structural hole in Figure VA but not in VB. According to Burt (2000), these structural holes “broker the flow of information between people, and control the projects that bring together people from opposite sides of the hole.” A key part of this argument is that low-closure networks provide easier access to small favors, advice, information, and other resources.

To explore these issues in the social collateral model, we first develop a measure of network closure, building on the idea that high closure is associated with having multiple paths to a smaller set of agents. We begin by counting the total number of paths of an agent, using the concept of network flows. Fix a network with integer-valued capacities  $c$ ; then the network flow  $T^{st}(c)$  is effectively the number of disjoint paths of unit capacity between  $s$  and  $t$ . Thus, the total path number for  $s$  is simply  $T^s(c) = \sum_{t \in W} T^{st}(c)$ .

In Figure V,  $s$  has a total of four paths in both networks; the difference in closure comes from the fact that in VA, these four paths reach four different people, whereas in VB they reach only

two people, but there are two paths connecting  $s$  with either of them.<sup>16</sup> To generalize this observation, let  $P^s(n)$  denote the share of paths  $s$  has with agents to whom he has at least  $n$  paths, so that  $P^s(2) = 0$  in Figure VA and  $P^s(2) = 1$  in Figure VB.<sup>17</sup> Clearly,  $P^s(0) = 1$  always, and  $P^s(n)$  is nonincreasing in  $n$ .

**DEFINITION 5.** The network neighborhood of  $s$  has a higher closure than the neighborhood of  $s'$  if

- (i)  $T^s(c) = T^{s'}(c)$  so that  $s$  and  $s'$  have the same total number of paths; and
- (ii) For each  $n$ ,  $P^s(n) \geq P^{s'}(n)$ , so that a greater share of paths connect  $s$  to people with whom he has many paths.

These conditions imply that if the neighborhood of  $s$  has higher closure, then  $s$  is connected to fewer people through many paths.<sup>18</sup>

This definition allows us to compare high- and low-closure neighborhoods. The key theoretical insight is that higher closure increases trust but reduces access. For example, in Figure VB, two people trust  $s$  with assets of value  $V \leq 2$ ; although access is low, trust is high in this closed network. In contrast, in Figure VA,  $s$  can borrow from four people, but the asset value can be at most 1: access has increased, but at the cost of a reduction in pairwise trust. Due to this trade-off, whether high or low closure is associated with greater welfare depends on what assets are exchanged: trust is more important for high-value assets whereas access matters more for low-value assets.

To formalize this trade-off between access and pairwise trust, we let  $f(v)$  denote the density of  $F(v)$  and let  $\tilde{\omega}(V) = f(V)\omega(V)$ , the frequency-weighted profits from the ability to borrow  $V$ . Note that  $\tilde{\omega}(V)$  depends both on the probability that an asset of value  $V$  is needed ( $f(V)$ ), and on the profits this asset generates ( $\omega(V)$ ). We say that the economy is a *high-value exchange environment* if  $\tilde{\omega}(V)$  is increasing: in this case high-value transactions generate greater welfare  $\tilde{\omega}(V)$ , either because they are more likely or because they are more productive. Conversely, we

16. To see why  $s$  has four paths in Figure VB, note that there are two paths connecting  $s$  to  $t_1$ , the direct one and the indirect one through  $t_2$ ; and similarly, two paths connect  $s$  to  $t_2$ .

17. If arrangements are limited by a circle of trust, then  $T^s(c)$  and  $P^s(n)$  need to be computed in the corresponding subgraph of permissible links.

18. Also note that (ii) is equivalent to requiring that the cumulative distribution function  $1 - P^s(\cdot)$  first-order stochastically dominates  $1 - P^{s'}(\cdot)$ .

say we are in a *low-value exchange environment* when  $\tilde{\omega}(V)$  is decreasing.

**PROPOSITION 2.** In a high-value exchange environment, a neighborhood with higher closure leads to a higher expected payoff to  $s$ . Conversely, in a low-value exchange environment, a neighborhood with higher closure leads to a lower expected payoff to  $s$ .

In a low-value exchange environment, the access provided by low closure is more attractive, because knowing more people directly or indirectly increases the likelihood that  $s$  can obtain a low-value asset. This logic is in line with Granovetter's and Burt's basic argument about the strength of weak ties and the benefits of a dispersed social network in providing access to assets with low moral hazard, such as small favors, information, or advice.<sup>19</sup> In contrast, in a high-value exchange environment, closure is better. Here, a reduction in access is more than compensated for by the fact that, through his dense connections,  $s$  will be able to borrow even high-value assets. This finding parallels Coleman's general argument for network closure, and particularly his example of the wholesale diamond market in New York City, where the exchange of valuable stones requires high trust between dealers.<sup>20</sup>

The results of Proposition 2 are related to Putnam's (2000) concepts of bridging and bonding social capital. In Putnam's view, bonding social capital is associated with dense social networks and is good for generating reciprocity between agents who know each other well. In contrast, the networks underlying bridging social capital are "outward looking and encompass people across diverse social cleavages," and are good for "linkage to external assets and for information diffusion." These two concepts parallel our distinction between trust and access; our results thus provide formal foundations as well as network-based measures for bonding and bridging social capital.

*Community Size and Network Closure.* What determines network closure? In Allcott et al. (2007), we argue that in practice, community size should be an important determinant. The

19. Section IV.B develops a variant of our basic setup where exchange of information is explicitly modeled.

20. Vega-Redondo (2006) reports a related finding in a model of repeated games played in networks. He shows that stability of cooperative behavior depends on a certain measure of network cohesiveness.

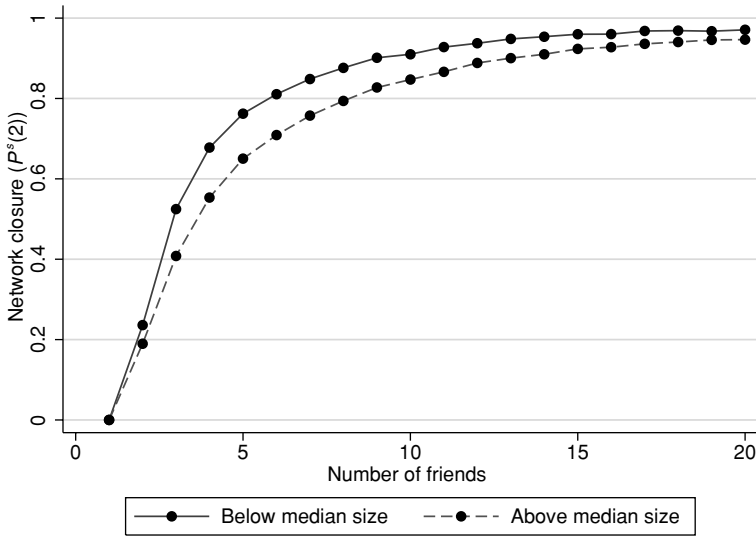


FIGURE VI  
Community Size and Network Closure

The figure is taken from Allcott et al. (2007). The figure plots average network closure for students by the number of their friends for schools below median size (solid line) and schools above median size (dashed line). For each student  $s$ , closure is measured as  $P^s(2)$ , the share of paths  $s$  has to others with whom he has at least two paths, within the circle of trust that includes links up to distance 2 from  $s$ . See Definition 5 and Section IV.A for details. The figure is constructed using data from 142 U.S. middle and high schools in the National Longitudinal Study of Adolescent Health; observations with number of friends greater than 19 were excluded (less than 1% of total).

intuition is straightforward: in a small community, the pool of potential friends is limited, which makes it more likely that two agents share common friends. In Allcott et al. (2007), we confirm this intuition using data on the social networks of students in the National Longitudinal Study of Adolescent Health (AddHealth).<sup>21</sup> Normalizing all link capacities to unity, we build on Definition 5 to measure the closure of the network around a student  $s$  with  $P^s(2)$ , the share of all paths that  $s$  has that connect him or her with others with whom he or she is connected through at least two paths.<sup>22</sup> This quantity is always between zero and one, and higher values represent more closed networks. Figure VI compares this measure

21. AddHealth is a representative sample of 142 U.S. public and private middle and high schools in 1994 and 1995.

22. We also restrict the “circle of trust” to links that are within distance 2 from agent  $s$ . The distance of a link  $(u, v)$  from  $s$  is the arithmetic average of the length of the shortest paths connecting  $s$  to  $u$  and  $s$  to  $v$ .

of closure for schools below and above the median size, for each possible value of a student's number of friends. This figure confirms that community size is an important predictor of closure in practice: even holding fixed a student's number of friends, smaller communities exhibit higher network closure.

*Implications for Organizations.* The connection between community size and closure, combined with Proposition 2, has implications for organizational design. In environments where access to small favors such as providing information is important, communities should be larger. This can be achieved through a flat organizational structure where rank does not limit interactions. For example, academic communities in the United States have a relatively informal culture, generating a large community of researchers; this encourages the development of weak ties and creates access to ideas. In contrast, organizations where trust is important can create it by having smaller communities. For instance, the hierarchical structure of armies limits interactions to peers of the same rank, creating networks with high closure and bonding social capital.

Our results also help explain the empirical fact that community size is often negatively correlated with prosocial behaviors such as volunteering, work on public projects, and helping friends (Putnam 2000). The traditional explanation is that in large communities people have fewer friends (Jacobs 1993). Our results suggest that even controlling for the number of friends, large communities have less dense social networks, which limits the provision of valuable public goods.

#### *IV.B. Job Search and Trust in Recommendations*

Sociologists have long recognized the importance of networks for finding jobs. For example, in *Getting a Job*, Granovetter (1974) documents that 56% of his sample of white-collar workers found employment through personal contacts. One possible explanation is that information about job openings often travels through friends and acquaintances. This logic forms the basis of Granovetter's (1973) "strength of weak ties" theory, formally modeled by Calvo-Armengol and Jackson (2004), which predicts that weak links to agents with whom one has few common friends are most useful for job search, because they provide access to otherwise unobtainable information. However, the evidence about the strength of weak ties is mixed. Studies in U.S. cities (Bridges and



Villemez 1986; Marsden and Hurlbert 1988) find that both weak and strong ties are important for job search. In Japan, Watanabe (1987) documents that small business employers screen applicants using strong ties. In China, Bian (1997, 1999) argues that the *guanxi* system of personal relationships allocates jobs using strong ties and paths.

Granovetter (1974) provides a second reason for the importance of connections: networks can generate trust in job recommendations. When there is asymmetric information about the skills of job candidates, offers are often made based on the opinions of trusted recommenders. In Granovetter's sample, such trusted referrals are common: in 60% of all jobs obtained through a network path of length 2 or more, the worker's direct contact had "put in a good word" for him. Because trusted referrals are more likely to come through strong ties, this logic can help explain why many empirical studies have found strong ties to be more important.

We now explore the implications of network-based trust for job search using the social collateral model.<sup>23</sup> Consider an employer  $t$  who needs to fill a vacancy. Potential employees are either high or low types; if hired, a high type generates total value  $S_H$  and a low type generates  $S_L$ , where  $S_H > S_L > 0$ . In the formal labor market, worker types are unobservable, the proportion of high types is  $\pi_H$ , and the prevailing market wage rate is  $w$ . Thus, hiring from the labor market generates an expected surplus  $\bar{S} = \pi_H S_H + (1 - \pi_H) S_L$ , of which  $\bar{S} - w$  accumulates to the employer. However, the employer may be able to hire a known high type through his social network. If  $s$  is a high-type job candidate, and his type can be credibly communicated to the employer, then the surplus from hiring  $s$  versus hiring from the formal labor market is  $S_H - \bar{S}$ . Assuming that this surplus is divided by Nash bargaining, where the bargaining weight of the worker is  $\alpha$ , the wage of  $s$  if hired is  $w_H = w + \alpha \cdot (S_H - \bar{S})$ , and the excess profit of the firm relative to hiring from the labor market is  $(1 - \alpha) \cdot (S_H - \bar{S})$ .

Can the network credibly communicate the worker's type to the employer? To answer, assume that the type of worker  $s$  is only observed by himself and his direct friends, denoted  $s_1, \dots, s_k$ . Although these friends can, in principle, provide recommendations,

23. Saloner (1985) and Simon and Warner (1992) also study informal recommendations in labor markets. These papers set aside trust considerations by assuming that recommenders and firms have the same objective.

they face a moral hazard problem: a low-type worker  $s$  can bribe them to write good recommendations. Here bribes are interpreted broadly to include in-kind transfers, as well as being nice to the recommender. The amount candidate  $s$  is willing to spend on bribes is limited by the attractiveness of the job,  $\alpha \cdot (S_H - \bar{S})$ ; if he or she offers more, the bribes would exceed the profit from getting the job. This reasoning suggests that the network can only communicate worker type in a credible way when the employer's trust of recommenders,  $s_1, \dots, s_k$  exceeds the highest bribe that the worker can pay,  $\alpha \cdot (S_H - \bar{S})$ .

To formalize these ideas, we modify the basic model as follows. First, we assume that prior to sending recommendations, agents agree on an informal transfer arrangement that is to be activated if the worker turns out to be a low type. This arrangement represents the understanding that recommenders will be held responsible for bad recommendations. Second, we introduce the concept of side deals with bribes, where agent  $s$  might propose a new transfer arrangement, together with a set of bribes to be paid to his friends,  $s_1, \dots, s_k$ , in exchange for their good recommendations.<sup>24</sup> Finally, we introduce an auxiliary network,  $G^\infty$ , where links between  $s$  and his friends,  $s_1, \dots, s_k$ , have infinite capacity, and  $\tilde{T}^{st}(c)$  denotes the trust flow between  $s$  and  $t$  in this network.

**PROPOSITION 3.** In an equilibrium robust to side deals with bribes, low-type workers are never hired through the network. If and only if  $\tilde{T}^{st}(c) \geq \alpha \cdot (S_H - \bar{S})$ , there exists an equilibrium robust to side deals with bribes where a high-type worker  $s$  is hired.

The result simply states that when network-based trust between the employer and recommenders exceeds the sensitivity of profits to worker type, as measured by the term  $\alpha \cdot (S_H - \bar{S})$ , the true type of the worker can be credibly communicated. Several implications about networks and labor markets follow. (1) Network-based trust should be more important for high-skilled jobs, where the employer's profits are more sensitive to worker type. Proposition 2 then predicts a trade-off between weak and strong ties: for low-skill jobs, where type matters less, weak connections are best because they maximize access; but for high-skilled jobs, recommendations through strong links embedded in a dense network are more useful. (2) Jobs obtained through the network should

24. The formal details of these modifications are presented in Appendix I.

earn higher wages than jobs obtained in the market. Simon and Warner (1992) obtain the same prediction, but their mechanism is different: in their work, networks reduce uncertainty about the quality of the match, increasing the reservation wage; in contrast, in our model only high types are hired through the network. (3) Due to the increased importance of trust for high-quality jobs, the wage differential between network-based and market-based hires,  $w_H - w = \alpha \cdot (S_H - \bar{S})$ , should be positively related to skill intensity. (4) When filling high-skill vacancies, employers should search more through their networks.

These predictions are consistent with several empirical facts. The first prediction helps explain the mixed evidence about the strength of weak ties by showing that for many jobs strong ties should be more important; it also implies that the strength of weak ties should vary with the skill intensity of the job, a prediction that awaits empirical testing. Consistent with the second prediction, Granovetter (1974) reports that in his sample, "jobs offering the highest salary are much more prone to be found through contacts than others: whereas less than half of jobs yielding less than \$10,000 per year were found by contacts, the figure is more than three-quarters for those paying more than \$25,000." This positive correlation between referrals and salary is also confirmed by Gorcoran, Datcher, and Duncan (1980) and Simon and Warner (1992). Regarding the intensity of network search, Brown (1967) finds that among college professors, personal networks are more frequently used in obtaining jobs of higher rank, smaller teaching loads, and higher salaries and at more prestigious colleges. For these attractive jobs, reducing asymmetric information is likely to be more important, and hence, employers have a stronger preference for searching through their networks.

Our predictions would not emerge in a model where the network served purely as a source of information about job vacancies. In such an economy, the network does not reduce information asymmetries; hence the wage differential is zero and the importance of network-based recommendations does not vary with the type of the job. Our results thus suggest that a full analysis of networks in labor markets should incorporate both information transmission and trust in recommendations.

*Trust and Asymmetric Information.* The social collateral model can also be used to study other situations involving asymmetric information. For example, a simple alteration of our job

search framework shows that network-based recommendations can help identify whether a given borrower is intrinsically a trustworthy type.<sup>25</sup> A similar logic applies for transactions of valuable assets such as houses, which involve a potential “lemons” problem: sellers with whom the buyer has a high trust flow are more likely to be honest about the quality of the good, to avoid future retribution through social sanctions.<sup>26</sup> We conclude that the implications of social collateral in the presence of asymmetric information are similar to the basic model with moral hazard: higher trust flow can secure transactions where there is greater exposure to asymmetric information.

### V. MEASURING SOCIAL COLLATERAL IN PERU

We now empirically evaluate the social collateral model using a unique data set from two low-income Peruvian shantytown communities, collected by Dean Karlan, Markus Mobius, and Tanya Rosenblat, further described in Karlan et al. (2008). Two key features of these data make them particularly useful for our purposes: (1) information on the social networks of individuals and (2) data on informal loans between friends, relatives, and acquaintances.

#### V.A. *Data Description*

In 2005, a survey was conducted in two communities located in the Northern Cone of Lima. The heads of households and spouses (if available) in 299 households were interviewed. The survey consisted of two components: a household survey and a social network survey. The household survey recorded a list of all members of the household and basic demographic characteristics, including sex, education, occupation, and income; summary statistics for these variables are reported in Table I. Average monthly household income in the two communities was 957 and 840 Peruvian new soles (S/.), respectively, which equals approximately 294 and 258 US\$, using the exchange rate in 2005.

The social network component of the survey asked the household head and spouse to list up to ten individuals in the community

25. Karlan (2005) documents evidence that there is variation in individuals' trustworthiness, which is predictive of their financial behavior.

26. In line with this prediction, in the 1996 General Social Survey, 40% of home purchases and 44% of used car purchases involved a direct or indirect network connection between the buyer and seller or realtor (DiMaggio and Louch 1998).

TABLE I  
SUMMARY STATISTICS FOR TWO SHANTYTOWN COMMUNITIES IN PERU

Demographic variables	Mean	Standard dev.	Social network variables	Mean	Standard dev.
Female	0.50	0.50	Number of contacts	8.60	4.15
Age	35.84	14.37	Share of "neighbors"	0.59	0.49
Secondary ed.	0.71	0.21	Share of "friends"	0.39	0.49
Household inc.(S/.)	887.39	1,215.74	Share of "relatives"	0.02	0.15
Business-owner	0.20	0.40	Avg. size of loan (S/.)	75.88	121.20
			Geographic dist.	41.16	49.17

*Note.* The table shows summary statistics for adults (age at least 18). Income and loan amounts are reported in Peruvian new soles (S/). The exchange rate at time of the survey was 3.25 S/ for one US\$. Network variables are calculated for the nondirected network where a pair of individuals are classified as connected if one of them names the other as a friend. Geographic distance is reported in meters.

with whom the respondent spent the most time in an average week. We use this data to construct an undirected "OR"-network, where two agents have a link if one of them names the other. Agents have, on average, 8.6 links, and the average geographic distance between connected agents is 42 and 39 m in the two communities; this is considerably less than the geographic distance between two randomly selected addresses, which is 132 and 107 m, respectively.<sup>27</sup> About 59% of relationships were classified by respondents as "vecino" (neighbor) and 39% as "amigo" or "compadre" (friend). The share of "relativos" was just 2%.<sup>28</sup> Vecinos live slightly closer than amigos/compadres (35 versus 51 m). Over 90% of directly connected people met in the neighborhood for the first time.

Importantly for our purposes, the social network survey also recorded, for each responder, the set of friends from whom he or she had borrowed money during the previous twelve months. There were 254 informal loans in the data set; 167 borrowers in 138 households reported having borrowed on average 76 S/. (about 23 US\$) from 173 lenders during the past twelve months. Thus, informal borrowing is very common in these communities: 46% of all households have at least one household member who borrowed money in this manner. The mean age of both the borrower and the lender is 39 years and they live, on average, 36 m apart.

27. This is consistent with a body of work showing the importance of social distance in meeting friends, for example, Marmaros and Sacerdote (2006).

28. In the remainder of this section, we use the term "friend" for any network connection, whether vecino, amigo/compadre, or relativo.

### V.B. Empirical Framework

*Measuring Capacities and Trust Flow.* To adapt our model of social collateral to this empirical setting, we need to develop a measure of link capacity. We use the amount of time spent together as a proxy for the strength of a connection, capturing the intuition that link values depend on investment in joint social activity. In the data, the distribution of time spent together is skewed: the average responder spends less than six minutes with the bottom 10% of his/her friends and more than three hours with the top 10%. To obtain a more homogeneous measure, we define *normalized time* for two connected agents  $u$  and  $v$  as the value, for the amount of time they spend together, of the empirical cumulative distribution function of time spent together in their community. With this definition, the empirical distribution of normalized time  $\tau(u, v)$  across all connected pairs is a discretized uniform distribution on the unit interval in each community.

We assume that link capacities are created by an increasing production function  $g$  such that  $c(u, v) = g(\tau(u, v))$ ; that is, spending more time together results in stronger links. We compute the network flow between agents  $s$  and  $t$  by defining the circle of trust to be the subgraph that contains all links of  $s$  and  $t$ . This circle of trust allows a simple decomposition of the trust flow between  $s$  and  $t$  as

$$(6) \quad T^{st}(c) = g(\tau(s, t)) + \sum_{v \in N_s \cap N_t} g(\min(\tau(s, v), \tau(v, t))),$$

where the first term represents the *direct flow* and the second term is the *indirect flow*. Here  $N_s$  is the set of direct friends of agent  $s$ .

*Discrete Choice Framework.* A natural approach to estimating the social collateral model is to use observations on how much agents borrow, and to use the loan size as a lower bound for the trust flow. This approach runs into the difficulty that loan amounts are also affected by demand: a borrower might borrow less than the trust flow. To avoid explicitly modeling loan demand, we instead base our estimation on *who* the agent borrows from, exploiting the idea that people are more likely to borrow from friends who trust them. By conditioning on the borrower, this approach effectively controls for loan demand as a fixed effect.

We formulate the borrower's choice of lender as a discrete choice problem. Consider agent  $s$ , who is in need of a loan of size

$V$ , which he can borrow from potential lenders  $t_1, \dots, t_k$ . We write the total utility that  $s$  enjoys when he borrows from a particular lender  $t$  as

$$(7) \quad u_t = u(V, T^{st}(c) + \varepsilon_t),$$

where  $u$  is increasing and  $\varepsilon_t$  represents either measurement error in the trust between  $s$  and  $t$ , or a supply shock. Appendix II provides micro foundations for this representation by assuming that if  $V$  exceeds the level of trust  $T^{st}$ , the excess value must be secured using physical collateral that has some opportunity cost. Then, the borrower is more likely to turn to a lender who trusts him more, implying that

$$(8) \quad \text{preferred lender} = \arg \max_t [T^{st}(c) + \varepsilon_t],$$

because, conditional on the loan amount, (7) is maximized when trust is highest.

*Model Predictions.* We use the above discrete choice specification to test three predictions of the social collateral model. (1) Agents are more likely to borrow from friends with whom they have a stronger trust flow. This prediction is a direct implication of Theorem 1. (2) The contribution of an indirect path of a given strength is equal to the contribution of a direct link with the same strength. This prediction is made because there are no costs to including intermediate agents within the circle of trust in the borrowing arrangement. In a setup where the circle of trust is endogenized, as in Section III.F, the contribution of indirect paths would be smaller, but still positive. (3) Each indirect path contributes to borrowing through its weakest link. In particular, in decomposition (6), for each indirect  $s \rightarrow v \rightarrow t$  path, if we have  $\tau(s, v) < \tau(v, t)$ , then the contribution of the path to borrowing should only depend on  $\tau(s, v)$ .

Some of these predictions are consistent with alternative explanations. Time spent together can be correlated with the strength of altruistic feelings between the two agents and the ease with which information travels between them. Common friends can further strengthen altruism and information transmission. Trust flow can therefore be a proxy for the lender's altruism toward the borrower and the lender's ability to learn about the profitability of the borrower's project. There is no particular reason that in these alternative explanations the weakest link

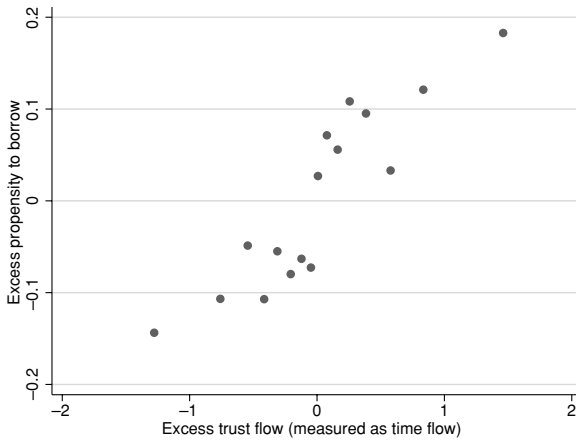


FIGURE VII

## Trust Flow and Borrowing

This figure is a residual plot, controlling for borrower fixed effects, of the relationship between trust flow, measured as time flow, and borrowing, where time flow is the sum of direct and indirect normalized time spent together. The figure is constructed as follows. For each borrower we calculate mean trust flow with all his or her friends, and define excess trust flow as the deviation from this mean. We similarly construct excess borrowing as the deviation from the average probability of borrowing across all friends. We sort all borrower–lender pairs by excess trust flow, group them into sixteen equal-sized bins, and plot the excess probability of borrowing (vertical axis) against the average excess trust flow (horizontal axis) for each bin.

should determine the strength of altruistic feelings or the strength of information transmission.<sup>29</sup> However, without better data, we cannot completely exclude these alternative explanations.

## V.C. Results

*Graphical Analysis.* We begin with a graphical analysis of trust flow and borrowing to highlight the basic patterns in the data. Assume that the strength of a link is proportional to normalized time:  $c(u, v) = c \cdot \tau(u, v)$ . Then trust flow  $T^{st}$  can be written as  $c \cdot \tau^{st}$ , where  $\tau^{st}$  measures the total (direct plus indirect) “time flow” between agents  $s$  and  $t$ , computed using equation (6).

Figure VII depicts the relationship between trust flow and borrowing in our sample, conditioned on borrower-specific fixed effects. The construction of the figure is the following. We introduce

29. One concrete model of altruism is where the lender cares about the utility of the intermediary who cares about the utility of the borrower. This model predicts that a geometric average of the two link values determines borrowing, which contradicts the weakest link condition of prediction 3. Similarly, if networks matter purely because they transmit information, then the average and not the minimum of link values should determine borrowing.



TABLE II  
BORROWING AS A FUNCTION OF DIRECT AND INDIRECT FLOW

Indirect time	Direct time	
	Above average	Below average
Above average	21.0%	42.0%
Below average	14.5%	22.5%

*Note.* This table shows the role of indirect paths in borrowing. Direct and indirect trust flow are computed as direct and indirect normalized time flow for each borrower and lender pair (see the notes to Figure VI or the text for details.) The construction of the table is as follows. We compute mean direct and indirect flow for each borrower by averaging across his/her friends, and create two indicator variables for whether direct and indirect flow is above or below the average. The table shows how loans are distributed across the resulting four bins (direct flow below or above average  $\times$  indirect flow below or above average).

an indicator variable  $I_{st}$ , which is one if we observe  $s$  borrowing from  $t$ . For each borrower  $s$  we calculate the mean time  $\bar{\tau}^s$  he or she spends with her friends, and the share  $\bar{I}_s$  of friends he or she borrows from. We then define the borrower's "excess time flow" with lender  $t$  as  $\tau^{st} - \bar{\tau}^s$ , and his or her "excess borrowing" from  $t$  by  $I_{st} - \bar{I}_s$ . Figure VII is simply a plot of excess borrowing against excess time flow, where observations are averaged over intervals of excess time flow to smooth out all uncorrelated noise. The figure shows a strong positive relationship, confirming the basic prediction that agents should be more likely to borrow from friends who trust them.

Figure VII does not distinguish between direct and indirect flows. To get a sense of the relative contribution of indirect paths, in Table II we group all friends of each borrower into four categories along two dimensions: whether the direct flow between borrower and friend is below or above the average direct flow, and whether the indirect flow between borrower and friend is below or above the average indirect flow. We then calculate the share of loans that fall into each of the resulting four categories. About 14.5 percent of loans involve borrower/lender pairs with both below-average direct flow and below-average indirect flow. Almost double as many loans involve borrower/lender pairs with either above-average direct or above-average indirect flow. About three times as many loans involve borrowers and lenders with both above-average direct and above-average indirect flow. Indirect paths appear to play an important role in creating social collateral for borrowing.

*Structural Estimation.* To analyze the relationship between trust flow and borrowing in greater detail, we now estimate the

discrete choice model (8). This allows us to measure the relative strength of different network links, as well as to formally test our predictions. We allow capacities to depend on the time spent together in a flexible way, by classifying every link as *weak*, *medium*, or *strong*, depending on whether the time spent together lies in the lowest, medium, or highest third of the time distribution for each of the two communities. Each direct and indirect path between borrower and lender then makes a weak, medium, or strong contribution to total flow, where the strength of these different link types is measured by unknown parameters  $c_W$ ,  $c_M$ , and  $c_S$ . Given our definition of the circle of trust, the trust flow  $T^{st}(c)$  between  $s$  and  $t$ , as given by (6), is easily seen to be a linear function of  $c = (c_W, c_M, c_S)$ . Assuming that the error term  $\varepsilon$  has the extreme value distribution, we can then estimate (8) as a conditional logit,

$$(9) \quad \Pr[\text{lender is } t] = \frac{\exp[(1/\lambda) \cdot T^{st}(c)]}{\sum_{u \in N_s} \exp[(1/\lambda) \cdot T^{su}(c)]},$$

where  $\lambda > 0$  measures the relative importance of the error term. Given the linearity of  $T^{st}$  in  $c$ , the unobserved parameters  $\lambda$  and  $c$  cannot be separately identified, but we can use the estimates to back out capacity ratios like  $c_S/c_M$ .

Table III reports our logit estimates. The first column contains our baseline specification; the coefficient estimates for total weak, medium, and strong flow correspond to  $c_W/\lambda$ ,  $c_M/\lambda$  and  $c_S/\lambda$  in the estimating equation. The effect of weak paths on borrowing is insignificant and small: gaining access to lenders through weak ties appears to be relatively less important for obtaining loans. Both medium and strong paths have a highly significant positive effect on borrowing, and the effect of strong paths is significantly greater. One additional medium path to a lender increases the probability of borrowing by a factor of 1.44, whereas an additional strong path increases the probability by a factor of 2.7. The ratio of the point estimates implies that the capacity of strong links is about three times as high as that of medium links:  $c_S/c_M \approx 2.7$ . These results support prediction 1, that trust flow should be positively related to borrowing, and highlight the importance of strong ties.

Is the contribution of an indirect path different from that of a direct path? To compare indirect and direct paths, in column (2) we add the number of indirect medium and strong paths as separate controls in the regression. According to our second prediction,

TABLE III  
TRUST FLOW AND CHOICE OF LENDERS CONDITIONAL LOGIT ESTIMATES

	(1)	(2)	(3)	(4)
Total weak flow ( $c_W/\lambda$ )	0.16 (0.143)	0.151 (0.142)	0.142 (0.164)	0.147 (0.164)
Total medium flow ( $c_M/\lambda$ )	0.365 (0.155)*	0.546 (0.266)*	0.341 (0.19)	0.543 (0.267)*
Total strong flow ( $c_S/\lambda$ )	0.991 (0.163)**	1.317 (0.283)**	0.988 (0.165)**	1.311 (0.284)**
Indirect weak flow		omitted		omitted
Indirect medium flow		-.190 (0.319)		-.226 (0.379)
Indirect strong flow		-.526 (0.363)		-.516 (0.368)
Weak-not weak flow			0.073 (0.313)	0.018 (0.315)
Medium-strong flow			0.069 (0.27)	0.06 (0.34)
Geographic distance	-.006 (0.003)*	-.006 (0.003)*	-.006 (0.003)*	-.006 (0.003)*
Obs.	988	988	988	988

*Note.* Each link is classified as *weak*, *medium*, or *strong* depending on whether the time spent together lies in the lowest third, medium third, or highest third of the time distribution. Weak, medium, and strong total flow are defined by noting that each direct and indirect path between borrower and lender makes either a weak, medium, or strong contribution to total flow. For indirect medium and strong flow we only count indirect paths. Weak-not weak flow counts paths where exactly one link is weak; medium-strong flow counts paths where one link is medium and one link is strong. We do not include indirect weak flow in columns (2) and (4) because we cannot separately identify total and indirect weak flow in our conditional logit estimation, as every potential lender has at least a weak link to the borrower.

\*5% significance level.

\*\*1% significance level.

the coefficients of these variables should be zero. We find that the estimated coefficients on indirect flow are negative, but not statistically significant, and smaller than the corresponding coefficients on total flow. These results show that both direct and indirect paths have a substantial positive effect on borrowing, confirming the basic intuition that dense networks are better in creating social collateral. The negative estimates of indirect flows, although insignificant, suggest that the effect of indirect paths is slightly smaller, which can be explained in our model by endogenizing the circle of trust as in Section III.F. Combined with the results about strong ties, these estimates suggest that dense networks and bonding social capital are important for obtaining loans in these communities.

We now test the prediction about the role of the weakest link in column (3), where we include two new explanatory variables

in the regression. “Weak-not weak flow” counts the number of indirect paths where one link is weak and the other is medium or strong, whereas “medium-strong flow” counts the number of paths where one link is medium and the other is strong. If prediction 3 is false, then these paths should have a positive effect on borrowing beyond what is predicted by the social collateral “weakest link” theory. The estimated coefficients on these variables are insignificant and small, providing strong evidence for the role of the weakest link in determining social collateral. These results are replicated in column (4), which includes the controls for indirect flows.

Our findings about the role of indirect paths and the weakest link property help distinguish our model from other explanations for borrowing, such as altruism and information transmission. One caveat with our econometric analysis is that if time spent together increases due to borrowing, reverse causality confounds the interpretation of the estimates. Thus, the evidence supports, albeit not exclusively, the social collateral model; moreover, strong ties and network closure, that is, bonding social capital, appear to be particularly important for borrowing. Importantly, the theoretical framework provides clear predictions that can be tested in further settings, with perhaps more control over key empirical identification issues.

## VI. CONCLUSIONS

This paper has built a model where agents use their social connections as collateral to secure informal loans. This model naturally leads to a definition of network-based trust, which we then use in applications related to network structure and welfare, trust in job search, and the measurement of social capital. We conclude by sketching three other applications of the social collateral model.

### VI.A. *Network Statistics*

When informal arrangements are restricted by the circle of trust to connections within a given social distance, our model generates a family of trust measures. Our working paper, Mobius and Szeidl (2007), shows that when all links have equal capacity, these measures are functions of several commonly used network statistics, including (1) number of friends; (2) the clustering coefficient, which is a measure of local network density; (3) the number of common friends of two agents; and (4) the number of transitive triples,

another measure of network density.<sup>30</sup> These results provide social collateral-based foundations for common network statistics.

### VI.B. Risk Sharing

Development economists often emphasize the importance of informal insurance in developing countries. Ambrus, Mobius, and Szeidl (2008) use the social collateral model to explore risk-sharing in networks. They find that good risk sharing requires networks to be expansive: larger sets of agents should have more connections with the rest of the community. Networks shaped by geographic proximity have this property, because agents tend to have friends at a close distance in multiple directions, helping to explain the observed good risk sharing in village environments. They also find that network-based insurance is local: socially closer agents insure each other more.

### VI.C. Dynamics of Trust and Panics

In the basic social collateral model, link capacities are exogenous. Mobius and Szeidl (2008) show that link values can be endogenized with multiple rounds of exchange. The strength of a relationship is, then, the sum of its direct value, as in the basic model, plus the indirect value, which derives from the ability to conduct transactions through the link in the future. In this framework, fluctuations can be amplified through a network multiplier similar to the social multiplier of Glaeser, Sacerdote, and Scheinkman (2003), because trust withdrawal that constrains exchange locally can lead to further trust withdrawals that ripple through the network. New technologies that limit future social interaction, such as television, can substantially reduce trust and social capital through this mechanism.<sup>31</sup>

## APPENDIX I: PROOFS

**DEFINITION 6.** A weak flow with origin  $s$  is a function  $g : W \times W \rightarrow \mathbb{R}$  with the following properties:

- (i) Skew symmetry:  $g(u, v) = -g(v, u)$ .
- (ii) Capacity constraint:  $g(u, v) \leq c(u, v)$ .
- (iii) Weak flow conservation:  $\sum_w g(u, w) \leq 0$  unless  $u = s$ .

30. These measures are used, for example, in Wasserman and Faust (1994), Watts and Strogatz (1998), Glaeser et al. (2000), and Jackson (2006).

31. In related work, Kranton (1996) and Spagnolo (1999) study the interaction between social and business activities.

A weak flow of origin  $s$  can be thought of as taking a certain amount from node  $s$  and carrying it to various other nodes in the network. By weak flow conservation, any node other than  $s$  receives a nonnegative amount.

LEMMA 1. We can decompose any weak flow  $g$  as

$$g = \sum_{u \in V} f_u,$$

where for each  $u$ ,  $f_u$  is an  $s \rightarrow u$  flow, that is,  $\sum_w f_u(v, w) = 0$  for all  $v \neq u$ ,  $v \neq s$ , and moreover  $\sum_w f_u(u, w) = \sum_w g(u, w)$ , that is,  $f_u$  delivers the same amount to  $u$  that  $g$  does.

*Proof.* Consider vertex  $u$  such that  $\sum_w g(u, w) < 0$ . By weak flow conservation, the amount of the flow that is left at  $u$  must be coming from  $s$ . Hence, there must be a flow  $f_u \leq g$  carrying this amount from  $s$ . With  $f_u$  defined in such a way, repeat the same procedure for the weak flow  $g - f_u$  with some other vertex  $u'$ . After  $f_u$  is defined for all vertices  $u$ , the remainder  $f'$  satisfies flow conservation everywhere and can be added to any of the flows. ■

Implicit summation notation: For a weak flow  $g$  and two vertex sets  $U \subseteq W$  and  $V \subseteq W$ , we use the notation that

$$f(U, V) = \sum_{u \in U, v \in V} f(u, v).$$

*Proof of Theorem 1. Sufficiency.* We begin by showing that when (4) holds, a side deal–proof equilibrium exists. By assumption, there exists an  $s \rightarrow t$  flow with value  $V$ . For all  $u$  and  $v$ , let  $h(u, v)$  equal the value assigned by this flow to the  $(u, v)$  link. Now consider the strategy profile where (1) the borrowing arrangement  $h$  is proposed and accepted, (2) the borrower returns the asset, and (3) all transfers are paid if the borrower fails to return the asset. This strategy is clearly an equilibrium. To verify that it is side deal–proof, consider any side deal, and let  $S$  denote the set of agents involved. For  $s$  to be strictly better off, it must be that he prefers not returning the asset in the side deal. Now consider the  $(S, T)$  cut. By definition, the amount that flows through this cut under the original arrangement is  $V$ ; but then the same amount must flow through the cut in the side deal, as well. This means that  $s$  must transfer at least  $V$  in the side deal; but then he cannot be better off. More generally, this argument shows that any transfer arrangement that satisfies flow conservation is side deal–proof.

*Necessity.* We now show that when (4) is violated, no side deal–proof equilibrium exists. We proceed by assuming to the contrary that a pure strategy side deal–proof equilibrium implements borrowing even though (4) fails. First note that on the equilibrium path, the borrower must weakly prefer not to default. To see why, suppose that the borrower chooses to default on the equilibrium path. Because the lender and all intermediate agents must at least break even, this implies that the borrower has to make a transfer payment of at least  $V$ . But then the borrower must weakly prefer not to default, because returning the asset directly has a cost of  $V$ . This also implies that all intermediate agents must have a zero payoff.

By assumption, there exists an  $(S, T)$  cut with value  $c(S, T) < V$ . We now construct a side deal where all intermediate agents in  $S$  continue to get zero, but the payoff of  $s$  strictly increases. The idea is easiest to understand in an equilibrium where promises are kept, that is, when all transfers satisfy the capacity constraint  $h(u, v) \leq c(u, v)$ . Then, we simply construct an arrangement that satisfies flow conservation inside  $S$  and delivers to the “boundary” of  $S$  the exact amount that was promised to be carried over to  $T$  under  $h$ . More generally, when the capacity constraints fail over some links, the deviation in the side deal can result in some agents in  $S$  losing friendships with agents outside  $S$ . To compensate for this loss, the side deal must deliver to the “boundary” of  $S$  an additional amount that equals the lost friendship value.

Formally, let  $g$  be a maximal  $s \rightarrow t$  flow and consider the restriction of  $g$  to  $S$ . This is a weak flow, and by the lemma it can be decomposed as  $g = \sum_{u \in S} g_u$ , where each  $g_u$  is an  $s \rightarrow u$  flow. Now for each  $u \in S$ , let  $g(u, T)$  and  $h(u, T)$  denote the amounts leaving  $S$  through  $u$  under  $g$  and  $h$ . Moreover, for each  $u \in S$ , let  $z(u, T)$  denote the total friendship value lost to  $u$  in the subgame where the borrower defaults, as a consequence of unkept transfer promises. Because  $g$  is a maximum flow and  $(S, T)$  is a minimal cut, it follows that  $g(u, T) \geq h(u, T) + z(u, T)$ . This is because any link between  $u$  and  $T$  is either represented in  $h(u, T)$ , if  $u$  pays the transfer, or  $z(u, T)$ , if  $u$  does not pay and loses the friendship. This inequality implies that, whenever  $h(u, T) + z(u, T) > 0$ , we also have  $g(u, T) > 0$ . As a result, we can define

$$h' = \sum_{u \in S} \frac{h(u, T) + z(u, T)}{g(u, T)} \cdot g_u.$$

Note that  $h'$  is a weak flow in  $S$  and delivers exactly  $h(u, T) + z(u, T)$  to all agents in  $S$ . Thus  $h'$  satisfies flow conservation within  $S$  and delivers to the “boundary” of  $S$  the sum of two terms:  $h(u, T)$ , which is the precise amount to be carried over to  $T$  under  $h$ , and  $z(u, T)$ , which is the loss of friendship  $u$  suffers due to not making other promised transfers. We claim that  $h'$  is a profitable side deal. First,  $h'$  satisfies all capacity constraints by construction. Second, all agents in  $S$  break even under  $h'$ , as they did in the original equilibrium. Third, the total value delivered by  $h$  is at most  $c(S, T) < V$ , which means that  $s$  pays less than  $V$  under  $h'$ , whereas he pays exactly  $V$  in the original equilibrium. We have constructed a side deal in which the borrower is better off and all other players are best-responding; hence, the original equilibrium was not side deal–proof.

*Proof for Section III.F. Transfer Constraints.* In this analysis, we use a more stringent equilibrium selection criterion: We look for equilibria where (i) all promised transfers are paid; and (ii) there are no profitable side deals. In the earlier analysis, there was no need to impose (i), because the characterization results showed that any level of borrowing that can be implemented can also be implemented using equilibria where all transfers are paid. With transfer constraints, requiring that all promises be credible has additional bite, because promises that are not credible can generate large punishment in the form of loss of friendship to agents who have small  $k_u$ . We find it plausible that such agents will not make promises that they know they cannot keep, but instead of providing formal micro foundations for this, we simply restrict ourselves to equilibria that are “credible,” in the sense that all promises are kept.

Consider the directed network  $G'$  defined in the text and let the maximum  $s_1 \rightarrow t_1$  flow in  $G'$  be denoted by  $T^{s_1 t_1}(c)$ .

**PROPOSITION 4.** There exists a side deal–proof equilibrium with credible promises that implements borrowing if and only if

$$(10) \quad V \leq T^{s_1 t_1}(c).$$

*Proof. Sufficiency.* If (10) holds, then take a flow with value  $V$ , and let the flow values between different agents define the transfer arrangement in our candidate equilibrium. Note that by construction, this borrowing arrangement satisfies the borrowing



constraints of all agents  $u$ . Moreover, the promised transfers in this arrangement will be kept because they all satisfy the capacity constraint. It remains to be shown that there are no profitable side deals; this follows from the same argument used in the proof of Theorem 1.

*Necessity.* Suppose that (10) fails, and consider an equilibrium where promised transfers are paid and borrowing is implemented. We now show that this equilibrium admits a side deal. Our argument is similar to the proof of Theorem 1, in that we build the side deal using a minimum cut on the network  $G'$ . However, the present setup has one additional difficulty: we need to make sure that the side deal emerging from the minimum cut does not separate agents from their duplicates.

Let  $(S', T')$  be a minimum cut. If for some  $u \neq s$  we have  $u_2 \in S'$ , then  $u_1 \in S'$  also holds, because  $u_2$  has only one incoming link, which originates in  $u_1$ . Let  $S$  be the union of  $s$  and the collection of agents  $u$  such that  $u_1 \in S'$ . We need to show that agents in  $S$ , as a group, do not have the right incentive to return the asset. To see why, consider first an agent  $u \in S$  such that  $u_2 \notin S'$ . It follows that the  $(S', T')$  cut separated  $u_1$  from  $u_2$ , by cutting the  $u_1 \rightarrow u_2$  link. But in this equilibrium, promises are kept, and, hence, the total obligation of  $u$  to agents outside  $S$  can be at most  $k_u$ , which is exactly the value of the cut link. Next consider an agent  $u \in S$  such that  $u_2 \in S'$ . For this agent, the total obligations to others outside  $S$  are bounded from above by the total value of the links originating in  $u_2$  that are cut. Summing over all  $u \in S$ , we conclude that the total obligations of all agents in  $S$  do not exceed the value of the  $(S', T')$  cut, and, hence, are strictly smaller than  $V$ . Thus,  $S$ , as a group, has an incentive to default. The actual side deal can now be constructed in the same way as in the proof of Theorem 1. ■

*Proof of Proposition 1.* Consider two capacities  $c_1 \leq c_2$ . Any flow between  $s$  and  $t$  that is feasible under  $c_1$  is also feasible under  $c_2$ ; hence the maximum flow cannot be lower under  $c_2$  than under  $c_1$ .

*Proof of Proposition 2.* We denote the share of total paths to agents with whom agent  $s$  has precisely  $j$  paths with  $q_s(j)$ . If we treat this function as a probability density function over the nonnegative integers, then an increase in closure is equivalent to a first-order stochastic dominance shift.

The expected payoff of  $s$ , conditional on his being the borrower, can be written as

$$\frac{1}{N} \sum_j \frac{q_s(j)}{j} \Pi(j) = \frac{1}{N} \sum_j q_s(j) \frac{\Pi(j)}{j},$$

which can be viewed as the expected value of the function  $\Pi(j)/j$  under the probability density  $q_s(j)$ . In a high-value exchange environment,  $\Pi(V)$  is convex because  $\Pi(V)' = \tilde{\omega}(V)$  is increasing; this, combined with the fact that  $\Pi(0) = 0$ , implies that  $\Pi(V)/V$  is nondecreasing. In this case, a first-order stochastic dominance increase in the probability density  $q_s(j)$  increases the expected payoff by definition. An analogous argument shows that in a low-value exchange environment, the same increase in the sense of first-order stochastic dominance reduces the expected payoff of  $s$ .

*Proof of Proposition 3. Preliminaries.* The timeline of the model with job search is the following. In stage 1, a set of agents, including  $s_1, \dots, s_k$  and  $t$ , agree on a transfer arrangement that specifies transfers  $h(u, v)$  to be made in the event that  $s_1, \dots, s_k$  send recommendations, and  $s$  is hired and then turns out to be a low type. In stage 2, agents  $s_1, \dots, s_k$  choose whether to recommend  $s$  to the employer  $t$ . In stage 3,  $t$  decides whether to hire  $s$  or not; profits are earned, and the type of  $s$  is publicly revealed. In stage 4, if needed, the transfer arrangement is executed; and in stage 5, agents consume the values of remaining links.

We consider a class of coalitional deviations that we call side deals with bribes. A side deal with bribes is a new transfer arrangement proposed by  $s$  to  $s_1, \dots, s_k$  and potentially some other agents at the beginning of stage 2, together with a set of bribes  $b_1, \dots, b_k$  that  $s$  pays to  $s_1, \dots, s_k$  in exchange for their recommendation. For simplicity, we assume that bribes are spot transactions: each agent  $s_j$  sends the recommendation at the same time that he receives the bribe.

We assume that when the surpluses from hiring through the network and in the market are the same,  $t$  always hires in the market.

*Proof.* Fix a pure strategy equilibrium robust to side deals with bribes. If a low type is hired in this equilibrium, then the expected surplus from the employment relationship is  $\bar{S}$ , which is the same as hiring in the formal market, and hence  $t$  never hires through the network. It follows that in equilibrium only high types

are hired in the network. Now suppose that in this equilibrium  $\tilde{T}^{st}(c) < \alpha \cdot (S_H - \bar{S})$  and the high-type worker is hired. Then the low type can propose a profitable side deal with bribes. As in the proof of the main theorem, this side deal includes all agents in a minimum cut separating  $s$  from  $t$  in  $G^\infty$  and transmits an amount equal to the maximum flow to agents at the boundary of the cut. The bribes in the side deal are specified to equal the amounts that flow through agents  $s_1, \dots, s_k$  in this flow. It follows that all agents weakly prefer accepting the side deal: intermediate agents at least break even by flow conservation, and the friends of  $s$  all break even because the bribes exactly compensate them for the payments to be made in the side deal. This contradiction shows that in any side deal-proof equilibrium where the high type is hired, we must have  $\tilde{T}^{st}(c) \leq \alpha \cdot (S_H - \bar{S})$ . Finally, if this inequality holds, then the transfer arrangement specified by the maximum flow in  $G^\infty$  is easily seen to be an equilibrium robust to side deals with bribes.

## APPENDIX II: MICRO FOUNDATIONS FOR SOCIAL SANCTIONS

In this Appendix, we develop a model where punishment at the level of the link arises endogenously. There are three key changes relative to the model presented in the main text: (1) with probability  $p > 0$ , the asset disappears, for example, is stolen by a third party, after the borrower uses it. (2) Each link “goes bad” with a small probability  $\varepsilon$  during the model, capturing the idea that friendships can disappear for exogenous reasons. (3) The utility of friendship is modeled using a “friendship game” where agents can choose to interact or stay away from each other. The payoffs of this friendship game depend on the capacity of the link and on whether the link has gone bad.

### A. Model Setup

This model consists of the following six stages:

*Stage 1: Realization of Needs.* Identical to stage 1 in Section III.

*Stage 2: Borrowing Arrangement.* In this model, there is uncertainty about whether the asset disappears after being used. As a result, the arrangement is now a set of state-contingent payments, where the publicly observable state of the world  $i$  is either  $i = 0$ , if the asset is returned, or  $i = 1$ , if the asset is reported stolen. A borrowing agreement consists of two parts. (1) A contract specifying payments  $y_i$  to be made by the borrower to the

lender in the two states ( $i = 0$  or  $1$ ). This contract can be thought of as a traditional incentive contract to solve the moral hazard problem in lending. If there were a perfect court system in the economy, then this contract would be sufficient to achieve efficient lending. (2) A *transfer arrangement* specifying payments  $h_i(u, v)$  to be made between agents in the social network if the borrower fails to make the payment  $y_i$ . Here  $h_i(u, v)$  denotes a payment to be made by  $u$  to  $v$  in state  $i$ .<sup>32</sup>

*Stage 3: Repayment.* If an arrangement was reached in stage 2, the asset is borrowed and  $s$  earns an income of  $\omega(V)$ , where  $\omega(\cdot)$  is a differentiable, nondecreasing function. Following the use of the asset, with probability  $p$  it is stolen. We assume that  $\omega(V) > pV$  for all  $V$  in the support of  $F$ , which guarantees that lending the asset is the socially efficient allocation. Even if the asset is not stolen, the borrower may choose to pretend that it is stolen and sell it at the liquidation value of  $\phi \cdot V$ , where  $\phi < 1$ . The borrower then chooses whether to make the payment  $y_i$  specified in the contract.

*Stage 4: Bad Links.* At this stage, any link in the network may go bad with some small probability. We think of a bad link as the realization by a player that he no longer requires the business or friendship services of his friend. As we describe below, cooperation over bad links in the friendship game is no longer beneficial. Therefore, agents who learn that a link has gone bad will find it optimal not to make a promised transfer along the link. From a technical perspective, bad links are a tool to generate cooperation without repeated play, just like the “Machiavellian types” in Dixit (2003) (see also Benoit and Krishna [1985]). In an equilibrium where promised transfers are expected to be paid, failure by  $u$  to make a payment will be interpreted by  $v$  as evidence that the link has gone bad. In this case,  $v$  will defect in the friendship phase, which reduces the payoff of the deviator  $u$  by  $c(u, v)$ .

To formalize bad links, assume that for every link of every agent, with a small probability  $\varepsilon > 0$  independent across agents and links, the player learns that his link has gone bad at this stage. Thus, for any link  $(u, v)$ , the probability that the link has not gone bad is  $(1 - \varepsilon)^2$ ; and for any link  $(u, v)$  where  $u$  does not learn that the link has gone bad,  $u$  still believes, correctly, that with probability  $\varepsilon$  the link has gone bad.

32. The circle of trust may restrict the links over which arrangements may be proposed. This case can be treated in the proof by assuming that  $G$  denotes the subgraph of permissible links.

*Stage 5: Transfer Payments.* If the borrower chose to make payment  $y_i$  in stage 3, then this stage of the game is skipped, and play moves on to the friendship phase. If the borrower did not make payment  $y_i$ , then at this stage agents in the social network choose whether to make the prescribed transfers  $h_i(u, v)$ . Each agent has a binary choice: either he makes the promised payment in full or he pays nothing.

*Stage 6: Friendship Game.* Each link between two agents  $u$  and  $v$  has a *friendship game* with an associated value  $c(u, v)$ . As long as the link is good, the friendship game is a two-player coordination game with two actions, with payoffs

		C	D
C	$c(u, v)$	$c(u, v)$	$0 \quad c(u, v)/2$
D	$c(u, v)/2$	$0$	$-1 \quad -1$

This game has a unique equilibrium (C,C) with payoff  $c(u, v)$  to both parties, which represents the benefit from friendly interactions. A party only derives positive benefits if his or her friend chooses to cooperate; and benefits are highest when there is mutual cooperation. If a link has gone bad, cooperation is no longer beneficial, and the payoffs of the friendship game change:

		C	D
C	$-1$	$-1$	$0 \quad 0$
D	$0$	$0$	$0 \quad 0$

Here, mutual cooperation leads to the low payoff of  $-1$ , capturing the idea that parties who are no longer friends might find it unpleasant to interact. If either party defects, the payoff of both parties is set to zero. The payoffs in the friendship game imply that if a player knows that a link has gone bad with probability 1, a best response is to play D.

*B. Model Analysis*

Because there is uncertainty in this model, we need to extend the concept of side deals to Bayesian games.

**DEFINITION 7.** Consider a pure strategy profile  $\sigma$  and a set of beliefs  $\mu$ . A side deal with respect to  $(\sigma, \mu)$  is a set of agents  $S$ , a transfer arrangement  $\tilde{h}_i(u, v)$  for all  $u, v \in S$ , and a set of continuation strategies and beliefs  $\{(\tilde{\sigma}_u, \tilde{\mu}_u) \mid u \in S\}$  proposed by  $s$  to agents at the end of stage 2, such that

- (i)  $U_u(\tilde{\sigma}_u, \tilde{\sigma}_{S-u}, \sigma_{-S} \mid \tilde{\mu}_u) \geq U_u(\sigma'_u, \tilde{\sigma}_{S-u}, \sigma_{-S} \mid \tilde{\mu}_u)$  for all  $\sigma'_u$  and all  $u \in S$ ,
- (ii) The beliefs  $\tilde{\mu}$  satisfy Bayes' rule whenever possible if play is determined by  $(\tilde{\sigma}_S, \sigma_{-S})$ ,
- (iii)  $U_u(\tilde{\sigma}_S, \sigma_{-S} \mid \tilde{\mu}_u) \geq U_u(\sigma_S, \sigma_{-S} \mid \mu)$  for all  $u \in S$ ,
- (iv)  $U_s(\tilde{\sigma}_S, \sigma_{-S} \mid \tilde{\mu}_u) > U_s(\sigma_S, \sigma_{-S} \mid \mu)$ .

The only conceptually new condition is (ii), which is clearly needed in a Bayesian environment. Motivated by this definition, our equilibrium concept will be a side deal–proof perfect Bayesian equilibrium.

**THEOREM 2.** There exists a side deal–proof perfect Bayesian equilibrium that implements borrowing between  $s$  and  $t$  if and only if the asset value  $V$  satisfies

$$(11) \quad V \leq T^{st}(c) \cdot \frac{(1 - \varepsilon)^2}{\phi + p(1 - \phi)}.$$

*Proof.* We begin by analyzing the optimal incentive contract in the absence of enforcement constraints. Suppose that  $s$  makes payments  $x_i$  ( $i = 0$  or  $i = 1$ ) in the two states of the world. What values of  $x_i$  guarantee that  $s$  chooses to return the asset and  $t$  breaks even? To prevent  $s$  from stealing, the excess payment if the asset is reported stolen must exceed the liquidation value  $\phi V$ :

$$(12) \quad x_1 - x_0 \geq \phi V.$$

For the lender to break even, he has to receive at least  $pV$  in expectation:

$$(13) \quad px_1 + (1 - p)x_0 \geq pV.$$

The minimum transfers that satisfy (12) and (13) are

$$(14) \quad x_0 = p(1 - \phi)V \quad \text{and} \quad x_1 = [\phi + p(1 - \phi)]V.$$

When the enforcement constraints are brought back, it is intuitive that borrowing can be implemented in the network as long as  $\max[x_0, x_1]$  does not exceed the maximum flow between  $s$  and  $t$ : in that case, the lender can just transfer  $x_i$  to the borrower along the network. Because  $x_1 > x_0$ , this requires that  $x_1$  not exceed the maximum flow, or equivalently

$$V \leq c(s, t) \cdot \frac{(1 - \varepsilon)^2}{\phi + p(1 - \phi)},$$

which is indeed the condition in the theorem. We now turn to the proof.

*Sufficiency.* We begin by showing that when (11) holds, a side deal–proof equilibrium exists. Let  $x_i$  be defined by (14) and let  $y_i = x_i$ . By assumption, there exists a flow with respect to the capacity  $c$  that carries  $x_1/(1 - \varepsilon)^2$  from  $s$  to  $t$ . For all  $u$  and  $v$ , define  $h_1(u, v)$  to be  $1 - \varepsilon$  times the value assigned by this flow to the  $(u, v)$  link. Similarly, let  $h_0(u, v)$  be equal to  $1 - \varepsilon$  times a flow that carries  $x_0/(1 - \varepsilon)^2$  from  $s$  to  $t$ . Now consider the strategy profile in which (1) the transfer arrangement  $(x_i, h_i)$  is proposed and accepted, (2) the asset is borrowed and returned unless stolen, (3) every agent  $u$  pays every promised transfer  $h_i(u, v)$  if necessary, unless he learns that his link with  $v$  has gone bad, and (4) all agents play  $C$  in the friendship game unless they learn that the link has gone bad, in which case they play  $D$ . This strategy profile  $\sigma$  generates beliefs  $\mu$ , and  $(\sigma, \mu)$  constitute a perfect Bayesian equilibrium. To see why, note that conditional on others making the transfer payments, it is optimal for  $s$  to make the payments  $y_i$  and not to steal the asset. Also, because  $h_i(u, v) \leq (1 - \varepsilon)c(u, v)$ , all agents find it optimal to make the transfer payments given beliefs. Finally, because on-path play never gets to the transfers, all intermediate agents are indifferent between accepting the deal and rejecting it. In fact, even if the transfers were used in one or both states on path, intermediate agents would still break even, because  $h_i$  are defined using flows.

We also need to verify that the equilibrium proposed here is side deal–proof. Consider any side deal, and let  $S$  denote the set of agents involved. Suppose that after the side deal, the borrower reports that the asset is stolen with probability  $p' \geq p$ . Let  $T$  be the complement of  $S$  in  $W$ , and consider the  $(S, T)$  cut. By definition, the expected amount that flows through the  $(S, T)$  cut in state  $i$  if  $y_i$  is not paid equals  $x_i$ . If the borrower never chooses to pay  $y_i$  in the side deal, he will have to make sure that at least  $p'x_1 + (1 - p')x_0$  gets to the cut in expectation. Because all intermediate agents must break even in expectation, this implies that  $s$ 's expected payments must be  $p'x_1 + (1 - p')x_0$  or more. Thus the side deal comes with a cost increase of  $(p' - p)[x_1 - x_0]$ . The increase in expected cost is easily seen to be the same if the borrower chooses to pay  $y_i$  in one or both states. The expected benefit of the side deal is  $(p' - p)\phi V$ . By equation (12) the expected benefit does not exceed the expected cost; the side deal is not profitable to  $s$ , which is a contradiction. Hence the original arrangement was side deal proof.

*Necessity.* We now show that when (11) is violated, no side deal–proof equilibrium exists. We proceed by assuming to the contrary that a pure strategy side deal–proof perfect Bayesian equilibrium implements borrowing even though (11) fails. For simplicity, we assume that the equilibrium proposed transfers  $h_i(u, v)$  are expected to be paid by all agents  $u$  in stage 5 if the borrower chooses not to pay  $y_i$  directly; that is, we only focus on equilibria where promises are kept. This condition is not necessary to obtain the result, but simplifies the proof somewhat. If this condition holds, then  $h_i(u, v) \leq (1 - \varepsilon)c(u, v)$  holds for all transfers proposed in equilibrium, because the amount by which  $u$  can expect to benefit from his friendship with  $v$  is at most  $(1 - \varepsilon)c(u, v)$ .

Let  $\chi_i = 1$  if in state  $i$  on the equilibrium path,  $s$  chooses not to pay  $y_i$ , and let  $\chi_i = 0$  otherwise.

*Case I.*  $\chi_0 = \chi_1 = 1$ . In this case, on the equilibrium path,  $y_i$  are never paid, and instead the transfer arrangements are always used. Define the expected transfer  $h = ph_1 + (1 - p)h_0$ . By the individual rationality of intermediate agents,  $h$  satisfies weak flow conservation, and therefore by the lemma can be decomposed as

$$h = \sum_{u \in V, u \neq t} f_u + h',$$

where  $f_u$  is  $s \rightarrow u$  flow and  $h' = f_t$ . In words, the  $f_u$  flows deliver the expected profits to the intermediate agents, whereas  $h'$  is an  $s \rightarrow t$  flow that delivers the expected payoff to the lender. Denote  $\sum_{u \neq t} f_u = f$ ; then  $f$  is a weak flow delivering the payments to all intermediate agents.

Our proof strategy will be the following. First, we take out the profits of all intermediate agents from the capacity  $c$  and the transfer  $h$ , essentially creating a “reduced” problem where intermediate agents are expected to break even. Then we construct a side deal for this simpler case using the maximum flow–minimum cut theorem, and finally, transform this into a side deal of the original setup.

Let  $c'(u, v) = c(u, v) - f(u, v)/(1 - \varepsilon)$  be a capacity on  $G$ . Note that any flow  $g'$  under  $c'$  can be transformed into a flow  $g = g' + f/(1 - \varepsilon)$  that satisfies the capacity constraints  $c$ . Consider the functions  $h'_i = h_i - f$ . It is easy to verify that  $h'_i/(1 - \varepsilon)$  satisfy the capacity constraints with respect to  $c'$  and that  $h' = ph'_1 + (1 - p)h'_0$ . Let  $(S, T)$  be a minimal cut of the directed flow network with capacity  $c'$ . By the maximum flow–minimum cut



theorem, there exists a maximum flow  $g$  in the network that uses the full capacity of this cut. By assumption, the value of the cut under  $h_1$  satisfies  $h_1'(S, T)/(1 - \varepsilon) \leq g(S, T) < x_1/(1 - \varepsilon)^2$ , which implies that  $(1 - \varepsilon)[h_1'(S, T) - h_0'(S, T)] < \phi V$  because  $(1 - \varepsilon)|h| \geq pV$ . In words, the value flowing through the minimal cut in the two states does not provide sufficient incentives not to steal the asset.

We now construct a side deal for the reduced problem. The idea is to construct a transfer arrangement that satisfies flow conservation inside  $S$  and delivers to the “boundary” of  $S$  the exact amount that was promised to be carried over to  $T$  under  $h'$ . With such an arrangement, all agents in  $S$  will break even in each state, and thus the incentives that applied to  $S$  as a group will apply directly to agent  $s$ . Because  $S$  as a group did not have the right incentives, with the side deal  $s$  will not have the right incentives either.

Formally, using the implicit summation notation, for each  $u \in S$ ,  $g(u, T)$ ,  $h_1'(u, T)$ , and  $h_0'(u, T)$ , let denote the amounts leaving  $S$  through  $u$  via the maximum flow  $g$ ,  $h_1'$ , and  $h_0'$ . Clearly,  $(1 - \varepsilon)g(u, T) \geq h_1'(u, T)$  and  $(1 - \varepsilon)g(u, T) \geq h_0'(u, T)$ . Now consider the restriction of  $g$  to the set  $S$ . This is a weak flow, and by the lemma it can be decomposed as  $g = \sum_{u \in S} g_u$ . Define  $h_1'' = \sum_{u \in S} (h_1'(u, T)/g(u, T)) \cdot g_u$  and  $h_0'' = \sum_{u \in S} (h_0'(u, T)/g(u, T)) \cdot g_u$ . Then  $h_1''$  and  $h_0''$  are both weak flows in  $S$ , they satisfy  $h_i'' \leq (1 - \varepsilon)c'$ , and they deliver exactly  $h_1'(u)$  and  $h_0'(u)$  to all  $u \in S$ . Thus  $h_i''$  satisfies flow conservation within  $S$ , and delivers to the “boundary” of  $S$  the amount promised to be carried over to  $T$  under  $h_1'$ , as desired. The total value delivered by  $h_i''$  is the value of the cut links under  $h_i''$ ; hence the amount that leaves  $s$  in the two states under  $h''$  satisfies  $(1 - \varepsilon)[|h_1''| - |h_0''|] < x_1 - x_0$ , that is, is insufficient to provide incentives not to steal the asset.

Now go back to the original network, and consider a side deal with all agents in the set  $S$ , where these agents are promised a transfer arrangement  $f + h_i''$ . This is just adding back the profits of all agents to the side deal of the reduced problem. With this definition, the new side deal satisfies the capacity constraints  $f + h_i'' \leq (1 - \varepsilon)c$  because  $h_i'' \leq (1 - \varepsilon)c' = (1 - \varepsilon)c - f$ . Second, all agents in  $S$  will be indifferent, because they get the same expected profits delivered by  $f$  (note that  $h''$  is a flow in both states and thus nets to zero state by state). The agents who have links that are in the cut are indifferent because  $h''$  is defined so that its inflow equals the required outflow for these agents. Third, the side deal

does not have enough incentives for  $s$  not to steal the asset, because  $|h'_1| - |h'_0| < \phi V / (1 - \varepsilon)$ . Moreover, if the original deal was beneficial for  $s$ , then so is the new deal. This is because the cost of the original deal was  $|f| + |h'|$ . The cost of the new deal if the borrower follows the honest asset-return policy is  $|f| + |h''|$ . But both  $h'$  and  $h''$  are flows, and they are equal on the  $(S, T)$  cut; hence they have equal values. Therefore, by following an honest policy, the borrower will have a cost equal to what he had to pay in the original deal. However, because the incentive compatibility constraint is not satisfied, the borrower is strictly better off always stealing the asset in the side deal. This argument shows that there exists a side deal in which the borrower is strictly better off, and all other players are best-responding; hence the original equilibrium was not side deal proof.

It remains to consider the cases where either  $\chi_0$  or  $\chi_1$  is equal to zero. In these cases, define the expected transfer payments as  $h = p\chi_1 h_1 + (1 - p)\chi_0 h_0$ . As above,  $h$  is a weak flow and thus  $f$ , the weak flow delivering the expected profits to all intermediate agents can be defined. Similarly, one can define  $c'$  and  $h'_i$ , and letting  $(S, T)$  be the minimal cut of  $c'$ ,  $h'_1(S, T) / (1 - \varepsilon) < x_1 / (1 - \varepsilon)^2$  must hold.

*Case II.*  $\chi_0 = 1$  and  $\chi_1 = 0$ . Then  $h = (1 - p)h_0$  and the decomposition  $h = f + h'$  yields  $h_0 = f / (1 - p) + h' / (1 - p)$ , so that  $h'_0 = h_0 - f = f \cdot p / (1 - p) + h' / (1 - p)$  is a weak flow, because it is a sum of two weak flows. It follows that  $|h_0| = |f| + |h'_0| \geq |f| + |h'_0(S, T)|$ . Therefore  $|f| + |h'_0| \leq |h_0|$ , because  $h'_0$  is a flow and  $h'_0 = h'_0$  on the  $(S, T)$  cut. Moreover, incentive compatibility requires  $y_1 - (1 - \varepsilon)|h_0| \geq \phi V$ , whereas the break-even constraint of the lender means that  $py_1 + (1 - p)(1 - \varepsilon)[|h_0| - |f| / (1 - p)] \geq pV$ . Combining these inequalities gives  $y_1 \geq x_1 + (1 - \varepsilon)|f|$ . Now consider the side deal  $h'_i + f$  defined as above. Because  $|h'_0 + f| \leq |h_0| \leq y_0 / (1 - \varepsilon)$  and  $|h'_1 + f| < x_1 / (1 - \varepsilon) + |f| \leq y_1 / (1 - \varepsilon)$ , the borrower will strictly prefer this arrangement to the previous one. Because all intermediate agents get net profits delivered by  $f$  in both states in the side deal, they are indifferent. Thus the proposed arrangement is indeed a side deal.

*Case III.*  $\chi_0 = 0$  and  $\chi_1 = 1$ . Here  $h_1$  is a weak flow, which must deliver less than  $x_1 / (1 - \varepsilon)$  to  $t$ , because by assumption  $x_1 / (1 - \varepsilon)$  is more than the maximum flow. Thus incentive compatibility fails with the original agreement; even without any side deal, the lender is better off not returning the asset.

*Case IV.*  $\chi_0 = 0$  and  $\chi_1 = 0$ . Here a valid side deal is to pay  $y_0$  in state zero and propose the transfer arrangement  $h'_1$  for state 1. All intermediate agents are indifferent because they were getting zero in the original arrangement, and because  $h'_1 < x_1/(1 - \varepsilon) \leq y_1/(1 - \varepsilon)$ , the expected payment in the side deal is strictly lower than in the original deal.

In the proof so far, we have only considered the case where the borrower does not steal the asset on the equilibrium path. If the equilibrium is such that the borrower always steals, then  $\min[(1 - \varepsilon)|h_1|, y_1] \geq V$  must hold. If  $\chi_1 = 1$ , then  $h_1/(1 - \varepsilon)$  is a weak flow with respect to capacity  $c$  that must transfer at least  $V/(1 - \varepsilon)^2$  to  $t$ . This leads to a condition on the maximum  $s \rightarrow t$  flow that is stronger than (11). If  $\chi_1 = 0$ , then a valid side deal is to propose the transfer arrangement  $h'_1$  for both states. As above, all intermediate agents are indifferent, and  $h'_1 < x_1/(1 - \varepsilon) \leq y_1/(1 - \varepsilon)$  holds, which proves that the expected payment in the side deal is strictly lower than in the original deal.

### APPENDIX III: EMPIRICAL MODEL

The utility function (7) that forms the basis of the discrete-choice model can be micro founded in the following way. Suppose that borrower  $s$  needs a loan of value  $V$  and needs to decide which of his friends to borrow from. Each potential lender  $t$  has an opportunity cost  $k(V) + v_S$  of providing the loan, where  $v_S$  is a supply shock unobserved to the borrower, which is independent across lenders. If the borrower chooses lender  $t$ , he is expected to repay both the value and the lender's full opportunity cost.<sup>33</sup>

Beyond the cost of a loan, the choice of lender is also influenced by the level of trust. We assume that the true level of trust between  $s$  and  $t$  is  $\alpha + T^{st}(c) + \varepsilon_M$ , where  $\alpha + \varepsilon_M$  reflects both measurement error in network-based trust and other sources of trust. When the expected repayment  $k(V) + v_S$  exceeds the level of social trust between borrower and lender, the excess amount must be secured using physical collateral. We assume that providing such physical collateral (e.g., a radio or a bicycle) has an opportunity cost that equals  $\gamma$  times the value of collateral. With these assumptions,

33. In many societies there is a social convention that agents are only to repay the nominal amount borrowed. However, there is often an understanding that lenders should be further compensated using in-kind transfers and gifts. Here we do not distinguish between these different forms of compensation.

the realized utility of borrowing from  $t$  is

$$\omega(V) - \gamma \cdot \max[0, k(V) + v_S - \alpha - T^{st}(c) - \varepsilon_M] - k(V) - v_S,$$

where  $\omega(V)$  is the utility from borrowing. In this expression  $v_S$  is unobservable, and hence  $s$  must take expectations over it. After taking expectations, we obtain

$$(15) \quad u(V, T^{st}(c) + \varepsilon_M)$$

for some  $u$  function that is strictly increasing in the second argument when  $v_S$  has full support.<sup>34</sup>

If we also incorporate observed supply shocks  $\bar{\varepsilon}_S$  into the analysis, then the final utility representation becomes

$$u(V, T^{st}(c) + \varepsilon_M - \bar{\varepsilon}_S) - \bar{\varepsilon}_S.$$

Assuming that  $u$  is close to linear in the second argument, which would be the case if  $v_S$  had sufficient variance, letting  $\varepsilon_S = (1 + 1/u_2)\bar{\varepsilon}_S$ , where  $u_2$  is the derivative of  $u$  in the second argument, we can approximate this total utility as a linear function of  $T^{st}(c) + \varepsilon_M - \varepsilon_S$ . In this representation, the error term captures a combination of supply shocks and measurement error in trust.

YALE UNIVERSITY  
HARVARD UNIVERSITY AND NBER  
IOWA STATE UNIVERSITY  
UNIVERSITY OF CALIFORNIA—BERKELEY

#### REFERENCES

- Ali, Nageeb, and David Miller, "Enforcing Cooperation in Networked Societies," U.C. San Diego, Working Paper, 2008.
- Allcott, Hunt, Dean Karlan, Markus Mobius, Tanya Rosenblat, and Adam Szeidl, "Community Size and Network Closure," *American Economic Review Papers and Proceedings*, 97 (2007), 80–85.
- Ambrus, Attila, Markus Mobius, and Adam Szeidl, "Consumption Risk-Sharing in Social Networks," Harvard University and U.C. Berkeley, Working Paper, 2008.
- Arrow, Kenneth J., *The Limits of Organization* (New York: W. W. Norton, 1974).
- Benoit, Jean-Pierre, and Vijay Krishna, "Finitely Repeated Games," *Econometrica*, 53 (1985), 905–922.
- Berg, Joyce, John Dickhaut, and Kevin McCabe, "Trust, Reciprocity and Social History," *Games and Economic Behavior*, 10 (1995), 122–142.
- Bian, Yanjie, "Bringing Strong Ties Back-In: Indirect Ties, Network Bridges, and Job Searches in China," *American Sociological Review*, 62 (1997), 366–385.
- , "Getting a Job through a Web of Guanxi in China," in *Networks in the Global Village: Life in Contemporary Communities*, B. Wellman, ed. (Boulder, CO: Westview Press, 1999).

34. Intuitively, if the trust flow is higher, there is a greater chance that the required repayment falls below it, in which case no physical collateral is needed.

- Bloch, Francis, Garance Genicot, and Debraj Ray, "Informal Insurance in Social Networks," *Journal of Economic Theory*, 143 (2008), 36–58.
- Bohnet, Iris, Benedikt Herrman, and Richard Zeckhauser, "The Requirements for Trust in Gulf and Western Countries," Harvard University and University of Nottingham, Working Paper, 2008.
- Bridges, William P., and Wayne J. Villemez, "Informal Hiring and Income in the Labor Market," *American Sociological Review*, 51 (1986), 574–582.
- Brown, David, *The Mobile Professors* (Washington, DC: American Council on Education, 1967).
- Burt, Ronald S., *Structural Holes: The Social Structure of Competition* (Cambridge, MA: Harvard University Press, 1995).
- , "The Network Structure of Social Capital," in *Research in Organizational Behavior*, No. 22, R. I. Sutton and B. M. Staw, eds. (Amsterdam: Elsevier Science, 2000).
- Calvo-Armengol, Antoni, and Matthew Jackson, "The Effects of Social Networks on Employment and Inequality," *American Economic Review*, 94 (2004), 426–454.
- Coleman, James S., "Social Capital in the Creation of Human Capital," *American Journal of Sociology*, 94 (1988), 95–120.
- , *Foundations of Social Theory* (Cambridge, MA: Harvard University Press, 1990).
- Cormen, Thomas H., Charles E. Leiserson, Ronald L. Rivest, and Clifford Stein, *Introduction to Algorithms* (Cambridge, MA: MIT Press, 2001).
- DiMaggio, Paul, and Hugh Louch, "Socially Embedded Consumer Transactions: For What Kind of Purchases Do People Most Often Use Networks?" *American Sociological Review*, 63 (1998), 619–637.
- Dixit, Avinash, "Trade Expansion and Contract Enforcement," *Journal of Political Economy*, 111 (2003), 1293–1317.
- Ellison, Glenn, "Cooperation in the Prisoner's Dilemma with Anonymous Random Matching," *Review of Economic Studies*, 61 (1994), 567–588.
- Fehr, Ernst, and Simon Gächter, "Cooperation and Punishment in Public Goods Experiments," *American Economic Review*, 90 (2000), 980–994.
- Fock, Henry K.Y., and Ka-shing Woo, "The China Market: Strategic Implications of Guanxi," *Business Strategic Review*, 9 (1998), 33–43.
- Ford, Jr., Lester Randolph, and Delbert Ray Fulkerson, "Maximal Flow Through a Network," *Canadian Journal of Mathematics*, 8 (1956), 399–404.
- Fukuyama, Francis, *Trust* (New York: Free Press, 1995).
- Glaeser, Edward L., David I. Laibson, José A. Scheinkman, and Christine L. Soutter, "Measuring Trust," *Quarterly Journal of Economics*, 115 (2000), 811–846.
- Glaeser, Edward L., Bruce I. Sacerdote, and Jose A. Scheinkman, "The Social Multiplier," *Journal of the European Economic Association*, 1 (2003), 345–353.
- Gorcoran, M., L. Datcher, and G. Duncan, "Information and Influence Networks in Labor Markets," in *Five Thousand American Families: Patterns of Economic Progress*, Vol. 8 (Ann Arbor: University of Michigan, Institute for Social Research, 1980).
- Goyal, Sanjeev, and Fernando Vega-Redondo, "Structural Holes in Social Networks," *Journal of Economic Theory*, 137 (2007), 460–492.
- Granovetter, Mark, "The Strength of Weak Ties," *American Journal of Sociology*, 78 (1973), 1360–1380.
- , *Getting a Job* (Cambridge, MA: Harvard University Press, 1974).
- , "Economic Action and Social Structure: The Problem of Embeddedness," *American Journal of Sociology*, 83 (1985), 481–510.
- Greif, Avner, "Contract Enforceability and Economic Institutions in Early Trade: The Maghribi Traders' Coalition," *American Economic Review*, 83 (1993), 525–548.
- Guiso, Luigi, Paola Sapienza, and Luigi Zingales, "Cultural Biases in Economic Exchange?" *Quarterly Journal of Economics*, 124 (2009), 1095–1131.
- Hardin, Russell, "The Street-Level Epistemology of Trust," *Analyse und Kritik*, 14 (1992), 152–176.
- Ioannides, Iannis M., and Linda Datcher Loury, "Job Information Networks, Neighborhood Effects, and Inequality," *Journal of Economic Literature*, 42 (2004), 1056–1093.

- Jackson, Matthew, "The Economics of Social Networks," in *Advances in Economics and Econometrics, Theory and Applications: Ninth World Congress of the Econometric Society*, Richard Blundell, Whitney Newey, and Torsten Persson, eds. (New York: Cambridge University Press, 2006).
- Jacobs, Jane, *The Death and Life of Great American Cities* (New York: Random House, 1993).
- Johnson, Simon, John McMillan, and Christopher Woodruff, "Courts and Relational Contracts," *Journal of Law, Economics and Organization*, 18 (2002), 221–277.
- Kandori, Michihiro, "Social Norms and Community Enforcement," *Review of Economic Studies*, 59 (1992), 63–80.
- Karlan, Dean, "Using Experimental Economics to Measure Social Capital and Predict Financial Decisions," *American Economic Review*, 95 (2005), 1688–1699.
- Karlan, Dean, Markus Mobius, Tanya Rosenblat, and Adam Szeidl, "Measuring Trust in Peruvian Shantytowns," Harvard University Discussion Paper, 2008.
- Knack, Stephen, and Philip Keefer, "Does Social Capital Have an Economic Payoff? A Cross-Country Investigation," *Quarterly Journal of Economics*, 112 (1997), 1251–1288.
- Kranton, Rachel E., "Reciprocal Exchange: A Self-Sustaining System," *American Economic Review*, 86 (1996), 830–851.
- La Porta, Rafael, Florencio Lopez-de-Silanes, Andrei Shleifer, and Robert W. Vishny, "Trust in Large Organizations," *American Economic Review Papers and Proceedings*, 87 (1997), 333–338.
- Lippert, Steffan, and Giancarlo Spagnolo, "Networks of Relations, Word-of-Mouth Communication, and Social Capital," SSE/EFI Working Paper in Economics and Finance 570, Stockholm School of Economics, 2006.
- Macaulay, Stewart, "Non-Contractual Relations in Business: A Preliminary Study," *American Sociological Review*, 28 (1963), 55–68.
- Marmaros, David, and Bruce Sacerdote, "How Do Friendships Form?" *Quarterly Journal of Economics*, 121 (2006), 79–119.
- Marsden, Peter V., and Jeanne S. Hurlbert, "Social Resources and Mobility Outcomes: A Replication and Extension," *Social Forces*, 66 (1988), 1038–1059.
- McMillan, John, and Christopher Woodruff, "Interfirm Relationships and Informal Credit in Vietnam," *Quarterly Journal of Economics*, 114 (1999), 1285–1320.
- Mobius, Markus, and Adam Szeidl, "Trust and Social Collateral," NBER Working Paper 13126, 2007.
- , "The Dynamics of Trust," Harvard University and U.C. Berkeley Working Paper, 2008.
- Putnam, Robert D., "Bowling Alone: America's Declining Social Capital," *Journal of Democracy*, 6 (1995), 65–78.
- , *Bowling Alone: The Collapse and Revival of American Community* (New York: Simon and Schuster, 2000).
- Saloner, Garth, "The Old Boys' Network as a Screening Mechanism," *Journal of Labor Economics*, 3 (1985), 255–267.
- Simon, Curtis J., and John T. Warner, "Matchmaker, Matchmaker: The Effect of Old Boy Networks on Job Match Quality, Earnings, and Tenure," *Journal of Labor Economics*, 10 (1992), 306–330.
- Spagnolo, Giancarlo, "Social Relations and Cooperation in Organizations," *Journal of Economic Behavior and Organization*, 38 (1999), 1–25.
- Standifird, Stephen S., and R. Scott Marshall, "The Transaction Cost Advantage of Guanxi-Based Business Practices," *Journal of World Business*, 35 (2000), 21–42.
- Townsend, Robert, "Risk and Insurance in Village India," *Econometrica*, 62 (1994), 539–591.
- Udry, Chris, "Risk and Insurance in a Rural Credit Market: An Empirical Investigation in Northern Nigeria," *Review of Economic Studies*, 61 (1994), 495–526.
- Uzzi, Brian, "Embeddedness in the Making of Financial Capital: How Social Relations and Networks Benefit Firms Seeking Financing," *American Sociological Review*, 69 (1999), 319–344.
- Vega-Redondo, Fernando, "Building Up Social Capital in a Changing World," *Journal of Economic Dynamics and Control*, 30 (2006), 2305–2338.

- Wasserman, Stanley, and Katherine Faust, *Social Network Analysis: Methods and Applications* (Cambridge, UK: Cambridge University Press, 1994).
- Watanabe, Shin, "A Comparative Study of Male Employment Relations in the United States and Japan," Ph.D. thesis, Department of Sociology, University of California at Los Angeles, 1987.
- Watts, Duncan J., and Steven H. Strogatz, "Collective Dynamics of 'Small-World' Networks," *Nature*, 393 (1998), 440–442.
- Wechsberg, Joseph, *The Merchant Bankers* (Boston: Little, Brown, 1966).